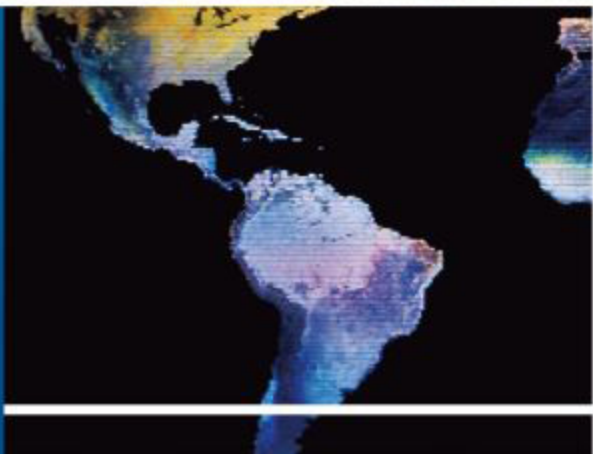
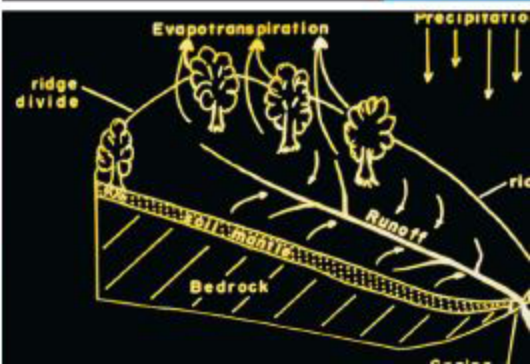


# PHYSICAL HYDROLOGY

Third Edition



S. Lawrence Dingman



# PHYSICAL HYDROLOGY

**Third Edition**

**S. Lawrence Dingman**

*University of New Hampshire*



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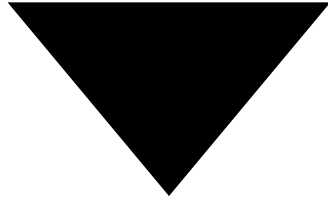
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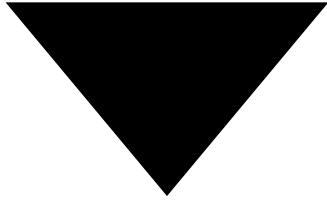
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# Preface

The goal of *Physical Hydrology* is to develop an understanding of the conceptual basis of the science of hydrology and to introduce the quantitative relations that implement that understanding in addressing scientific and water-resources-management questions. Previous editions apparently fulfilled a need for a comprehensive text in hydrology for upper-level undergraduates and graduate students, and I have been pleased with its reception by colleagues and students.

At the time of the first edition (1992), hydrology was still in the process of establishing itself as a distinct discipline with vital insights to fundamental and practical environmental problems (Eagleson et al. 1991), and the book was intended as a contribution to that process. By the time of the second edition (2002), much progress had been made in that regard, and my primary goals in revising *Physical Hydrology* were to incorporate significant advances in hydrologic science, to provide an explicit connection of that science to hydrologic modeling, and to make more complete and useful the treatment of the relation between scientific hydrology and water-resources management.

Hydrology is now well established as a distinct geoscience and, in the decade-plus since the second edition, there has been what seems to be exponential progress in the field. Much of this progress (which has been published in many dozens of different journals) is due to improvements in the ability to observe hydrologic variables and to assimilate and analyze large areally distributed data sets. But there has also been significant conceptual progress in understanding the ways in which the physics of micro-level processes relate (or don't relate) to the larger scales dictated by hydrologic questions and data availabil-

ity; in understanding the connections among hydrology, climate, ecosystems, soils, and geology; in understanding the nature and limitations of simulation models; and in developing new statistical techniques appropriate to the quantity and quality of hydrologic data.

It has been a daunting challenge to incorporate this progress in the third edition, and one that can be, at best, only partially met. In this attempt I have, in addition to making essential updates, made major changes in the organization and scope of the book: Former chapters 1–9 and portions of some appendices have been substantially reorganized into 10 chapters within three major sections, plus seven appendices, as described below.

---

## Part I: Introduction

### Chapter 1: Hydrology: Basic Concepts and Challenges

This chapter is a greatly expanded and much more substantive introduction to hydrologic science that now includes dimensions and units, properties of water, characteristics of hydrologic variables (including nonstationarity), and uncertainty in hydrology, as well as hydrologic systems and conservation equations. The chapter concludes with an application of many of the basic concepts in a case study exploring the prediction of watershed runoff.

### Chapter 2: The Global Context: Climate, Hydrology, and the Critical Zone

As in the previous edition, chapter 2 provides an overview of the global aspects of the hydrologic cycle

and its interactions with climate, soils, and major biomes. In addition to describing the basic features of global water and energy balances, this chapter incorporates many of the advances in these areas, particularly the role of teleconnections. It includes a detailed survey of the observational evidence for recent changes in hydroclimate and an analytical exploration of the impact of climate change on runoff.

---

## **Part II: Surface-Atmosphere Water and Energy Exchange**

### **Chapter 3: Principles and Processes**

This chapter provides the basic physical concepts for understanding precipitation, snowmelt, and evapotranspiration, which are covered in the following three chapters. It introduces relevant gas laws and the characterization of atmospheric water vapor, and describes the processes of precipitation formation and evaporation. The detailed treatment of turbulent diffusion in the lower atmosphere, which was formerly in an appendix, is now incorporated here.

### **Chapter 4: Precipitation**

The scope of this chapter is essentially the same as in the second edition, with updated treatments of the meteorology, measurement, areal estimation, and climatology of precipitation.

### **Chapter 5: Snow and Snowmelt**

This chapter also retains the scope of the previous editions, covering the hydrologic importance, material characteristics, measurement, and distribution of snow, as well as the physics and modeling of snowmelt.

### **Chapter 6: Evapotranspiration**

This chapter now occupies a more logical place in the sequence of topics, while retaining the basic treatment of the second edition. After reviewing the basic physics and classification of evaporative processes, it develops the basic approaches to estimating evaporation from water surfaces, bare soil, plants (interception and transpiration), and land surfaces, including the concepts of potential and reference-crop evapotranspiration.

---

## **Part III: Water Movement on the Land**

### **Chapter 7: Principles of Subsurface Flow**

This chapter provides the essential physics for understanding infiltration, ground-water flow, and runoff generation covered in the following chapters. It introduces the material properties of porous media and provides a more unified development of Darcy's law and the general equations of saturated and unsaturated subsurface flow than in the second edition. Capillarity is now discussed here as a basis for understanding water movement in the unsaturated zone. The moisture-characteristic and conductivity-characteristic curves, their analytic approximations, and their relations to soil texture are presented here in more detail than in the previous edition.

### **Chapter 8: Infiltration and Water Movement in Soils**

This chapter retains the treatment of the second edition, including water conditions in soils, qualitative and expanded quantitative descriptions of the infiltration process, the measurement of infiltration, infiltration over areas (now including scaling approaches), and redistribution of soil moisture.

### **Chapter 9: Ground Water in the Hydrologic Cycle**

As in the second edition, this chapter focuses on basic relations of regional ground-water flow to topography, geology, and climate and the connections between ground water and surface water in lakes, streams, and the ocean. The ground-water balance and approaches to estimating its components remain a central focus, and the discussion of base-flow separation has been expanded to include the base-flow index. As before, the chapter concludes with an introductory treatment of well hydraulics as a basis for understanding the effects of ground-water development on regional hydrology and the concept of "safe yield."

## Chapter 10: Runoff Generation and Streamflow

The contents of this chapter are essentially the same as in the second edition, but they have been reorganized to provide a more logical treatment. The chapter begins with a description of the watershed and its stream network and introduces the basic features of streamflow hydrographs and the geologic, topographic, meteorologic, and antecedent conditions that affect their shape. There is an expanded and updated discussion of chemical and isotopic end-member analysis for identification of runoff sources. The effects of channel processes on runoff characteristics are introduced. As in the second edition, the chapter concludes with an overview of conceptual rainfall-runoff models, including unit hydrographs and an updated treatment of the curve-number approach.

## Appendices

*Appendix A: Measurement Precision, Significant Figures, and Unit and Equation Conversion* is as in the second edition, except that the discussion of dimensions has been moved to chapter 1. This appendix now includes a table that can be used to make virtually any unit conversion that may arise in hydrology.

*Appendix B: Water as a Substance* is also similar to that in the second edition, but the introduction to some of water's unusual properties is also now incorporated in chapter 1. The appendix now includes an introduction to stable water isotopes and their use in hydrologic analysis.

*Appendix C: Statistical Concepts Useful in Hydrology* covers essentially the same material as in the second edition, except that (1) there is an expanded discussion of regional frequency analysis and (2) the discussion of model evaluation has been moved to appendix F. The tables and boxes for this appendix are included on the disk accompanying the text.

*Appendix D: Estimation of Daily Clear-Sky Incident Solar Radiation* is a revised and streamlined version of appendix E of the second edition. Material in the former appendix D (Water and Energy in the Atmosphere) is now incorporated in chapter 3.

*Appendix E: Stream-Gauging Methods for Short-Term Studies* is essentially the same as appendix F of the second edition.

*Appendix F: Hydrologic Simulation Modeling* is a revised version of material that was covered in chapter

2 of the second edition. It now concludes with an expanded presentation of quantitative criteria used for model calibration and validation (formerly treated in appendix C).

*Appendix G: Development of Scientific Hydrology* is an extensively revised overview of the history of scientific hydrology that concluded chapter 1 of the second edition.

## Other New Features

- Each chapter is accompanied by a number of exercises. These have been revised to emphasize analyses using material and data obtained from the World Wide Web and exploration of the local hydrologic environment.
- The disk accompanying the text has been revised, and in addition to providing Excel programs (including incident solar radiation, snowmelt, evapotranspiration, and infiltration) to use in conjunction with the exercises, it includes some longer explorations of lake water balances and the use of simulation modeling in exploring watershed hydrologic processes.
- SI units are now used exclusively.
- In keeping with the goal of providing an entrée to the literature of the field, this edition continues the practice of supporting its discussion with extensive reference citations, in the style of a journal article rather than that of most textbooks. In this revision, over 400 new reference citations have been added, and they now total over 1,100.
- Chapter 10 of the second edition of *Physical Hydrology* provided much valuable material on water-resources management. Although an extensive discussion of this topic was not included in the third edition, most of the second edition chapter 10 has been made available as an additional resource for students and instructors on the CD that accompanies the text.
- The detailed discussions of the way various hydrologic processes are simulated in the BROOK model have been dropped. Though the space previously devoted to describing the BROOK model as a window on simulation modeling had a justifiable purpose, omitting it seemed wise because (1) the model was not in widespread use and (2) there are many available models, and each instructor likely has her/his own preference that she/he may wish to use in conjunction with the text.

## Acknowledgments

Don Rosso, my editor at Waveland Press, first suggested that I undertake a third edition and has guided the process from the beginning. Extensive discussions with Richard H. Cuenca of Oregon State University and J. Matthew Davis of the University of New Hampshire, as well as the comments of three anonymous reviewers of the complete text, had a major influence on shaping the book's re-organization.

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Richard Vogel, W. Breck Bowden, James R. Wallis, Barry Keim, Mark Person, and C. Anthony Federer.

Diane Evans of Waveland Press has worked tirelessly to identify incorrect cross-references, missing literature citations, inconsistent symbols, and other editorial errors. I've corrected those, but am responsible for any that remain.

I am eternally grateful to Francis R. Hall, who was my mentor at UNH and did much to educate me about the science of hydrology and guide me through some of the pitfalls of academia. And, as with previous editions, I could not have completed this work without the love, support, and editorial assistance of my wife, Dr. Jane Van Zandt.

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Eastham, Massachusetts



# **Part I**

## Introduction





# Hydrology

## Basic Concepts and Challenges

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### 1.1 Definition and Scope of Hydrology

**Hydrology** is the geoscience that describes and predicts the occurrence and circulation of the earth's fresh water. The principal focus of hydrology includes:

- the distribution and movement of water substance on and under the earth's land surfaces, including its exchanges with the atmosphere;
- its physical and chemical interactions with earth materials; and
- the biological processes and human activities that affect its movement, distribution, and quality.

The circulation of water on land occurs in the context of the **global hydrologic cycle**, which includes the spatial and temporal variations of water substance in the oceanic and atmospheric as well as the terrestrial compartments of the global water system (figure 1.1). Thus, the study of the global hydrologic cycle is included in the scope of hydrology (Eagleson et al. 1991). The hydrologic cycle is a central component of the earth's climate system at all scales, from local to global (Peixoto and Oort 1992).

Figure 1.2 shows the major storage components and flows of the global hydrologic cycle, and figure 1.3 (on p. 5) shows the storages and flows of energy

and water that constitute the **land phase** of the cycle. Figure 1.4 (on p. 6) gives a quantitative sense of the range of time and space scales in the domain of hydrologic science.

Figure 1.5 (on p. 6) shows the position of hydrologic science in the spectrum from basic sciences to water-resource management. Hydrology is an inherently interdisciplinary geoscience, built upon the basic sciences of mathematics, physics, chemistry, and biology, and upon its sister geosciences. Much of the motivation for answering hydrologic questions has and will continue to come from the practical need to manage water resources and water-related hazards. Thus, hydrologic science is the basis for hydrologic engineering and, along with economics and related social sciences, for water-resources management.

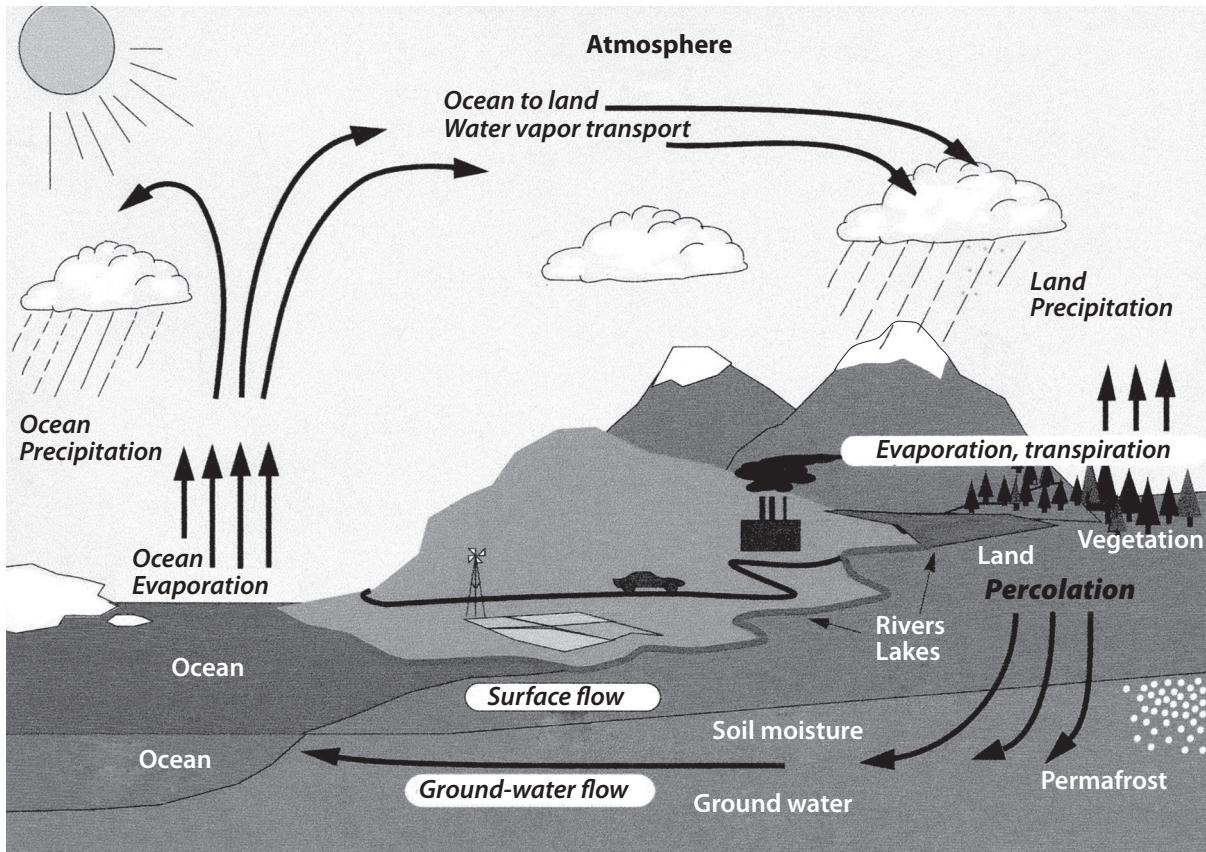
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### 1.2 Approach and Scope of This Book

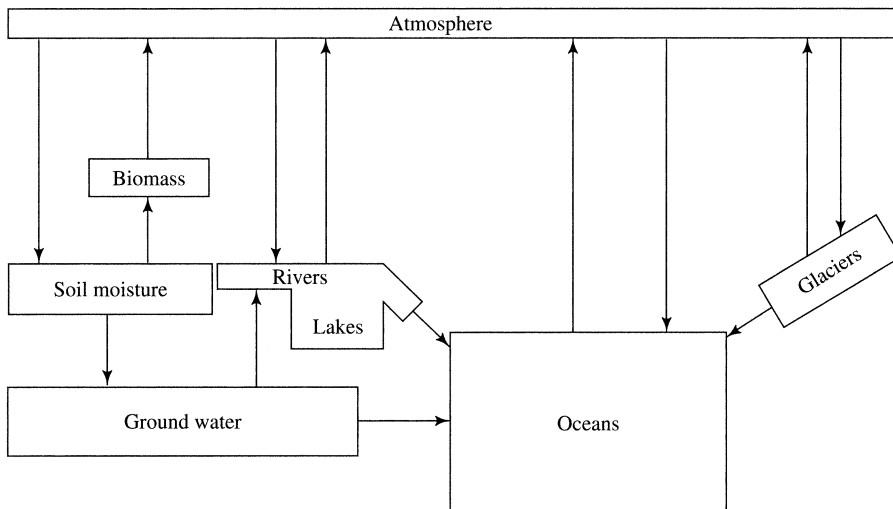
This text has three principal themes:

1. The basic physical concepts underlying the science of hydrology and the major conceptual and practical challenges facing it (chapters 1, 3, and 7).
2. The global scope of hydrologic science, including its relation to global climate, soils, and vegetation (chapter 2).





**Figure 1.1** Pictorial representation of the global hydrological cycle [Trenberth et al. (2007). Estimates of the global water budget and its annual cycle using observational and model data. *Journal of Hydrometeorology* 8:758–769, reproduced with permission of American Meteorological Society].



**Figure 1.2** The principal storages (boxes) and pathways (arrows) of water in the global hydrologic cycle.

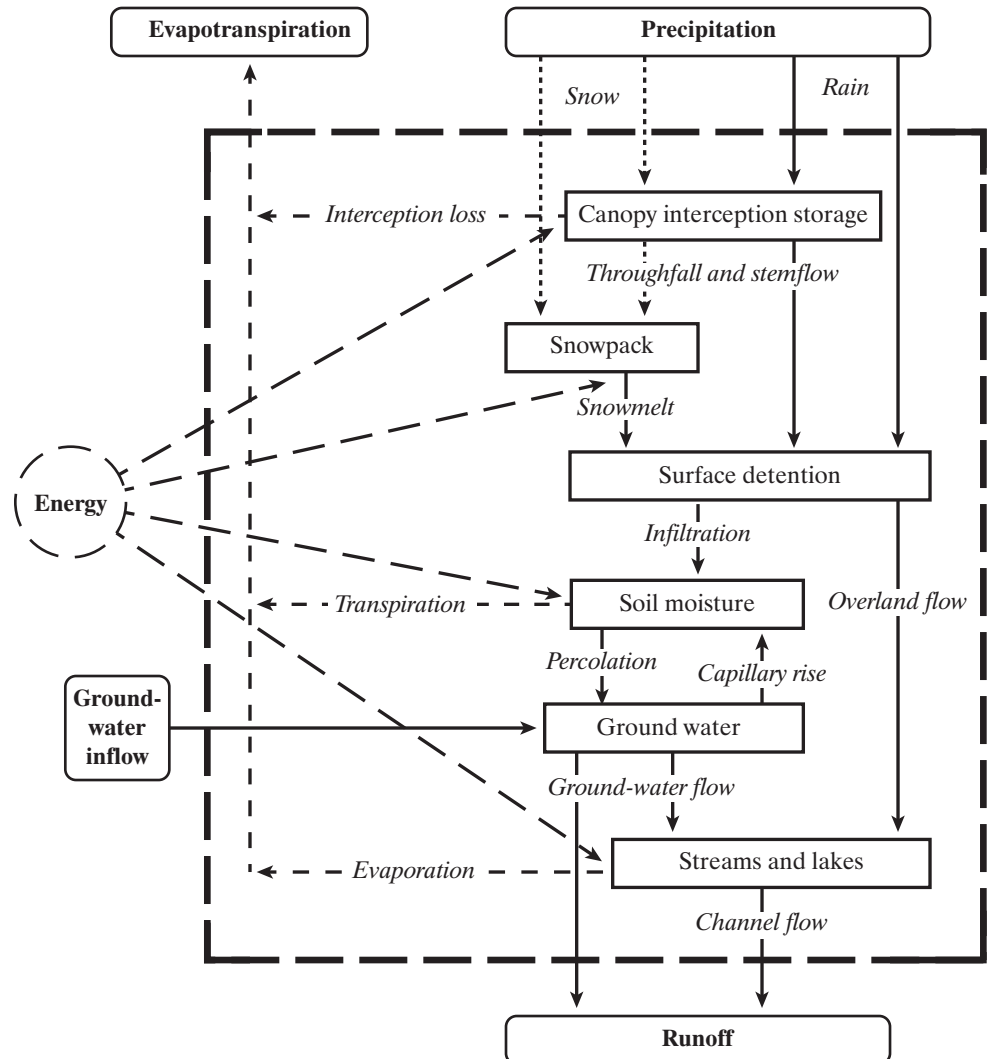
3. The land phase of the hydrologic cycle (chapters 4–6 and 8–10, which proceed more or less sequentially through the processes shown in figure 1.3).

A series of appendices supplement the main themes, including: (A) dimensions, units, and numerical precision; (B) properties of water; (C) statistical concepts; (D) computation of clear-sky solar radiation; (E) stream-gauging methods; (F) hydrologic modeling; and (G) the history of hydrology.

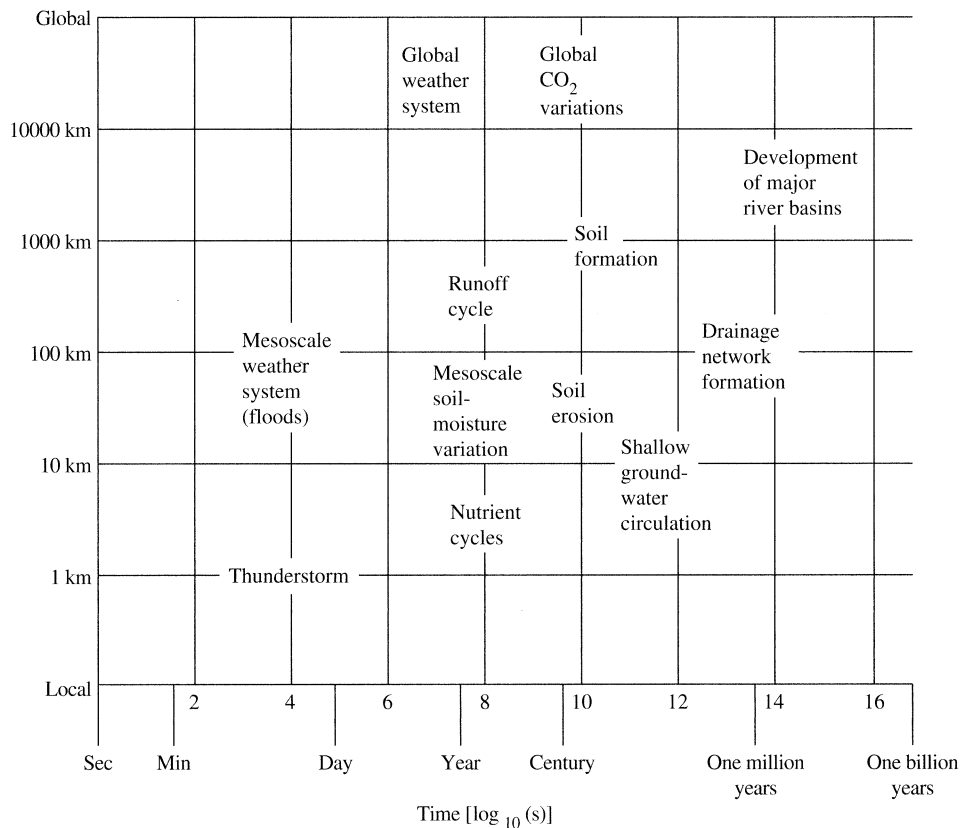
The treatments in chapters 3–10 draw on your knowledge of basic science (mostly physics, but also chemistry, geology, and biology) and mathematics to develop a sound intuitive and quantitative sense of the way in which water moves through the land phase of the hydrologic cycle. In doing this we focus on (1) relatively simple but conceptually sound quantitative

representations of physical hydrologic processes and (2) approaches to the measurement of the quantities and rates of flow of water and energy involved in those processes. Chapter 3 introduces the basic physical principles underlying the processes of precipitation formation, snowmelt, and evapotranspiration, which are covered in chapters 4–6. Chapter 7 introduces the basic physical principles underlying the movement of water in the subsurface, which are the foundation for understanding soil-water, ground-water, and runoff processes discussed in chapters 8–10.

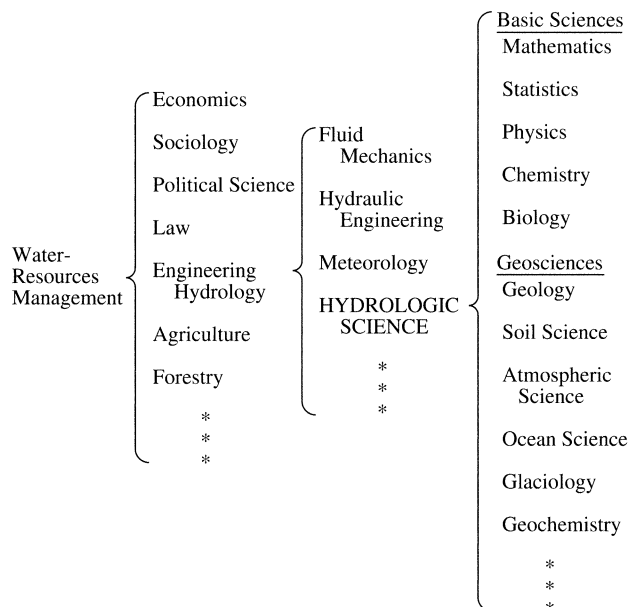
The material covered in this text constitutes the foundation of physical hydrology; the advances in the science that come in the next decades—in understanding watershed response to rain and snowmelt, in forecasting the hydrologic effects of land-use and



**Figure 1.3** The principal storages (boxes) and pathways (arrows) of water in the land phase of the hydrologic cycle. The heavy dashed line represents the boundary of a watershed or other region.



**Figure 1.4** Range of space and time scales of hydrologic processes (Eagleson et al., *Opportunities in the Hydrologic Sciences* © 1991 by the US National Academy of Sciences. Reprinted with permission of the National Academy Press).



**Figure 1.5** Hydrologic science in the hierarchy from basic sciences to water-resources management [adapted from Eagleson et al. (1991)].

climatic change over a range of spatial scales, in understanding and predicting water chemistry, and in other areas—will be built upon this foundation.

### 1.3 Physical Quantities and Laws

Hydrology is a quantitative geophysical science and, although it is not a fundamental science in the sense that physics and chemistry are, its basic concepts are founded on physical laws. Hydrological relationships are usually expressed most usefully and concisely as mathematical relations among hydrologic quantities, and familiarity with mathematics at least through calculus is required to understand and express hydrological concepts. In many practical and scientific problems, the essential mathematical relations involve statistical concepts, which are often somewhat theoretical and abstract; the basic statistical concepts frequently applied in hydrology are summarized in appendix C.

In this chapter we distill concepts from physics, statistics, and mathematics that are so frequently applied in hydrology that they can be considered basic hydrological concepts. In doing this, we will encounter a number of basic challenges that hydrologists face in pursuing their science. These problems arise because of the scale and complexity of hydrologic processes, difficulties of measurement (important quantities like evapotranspiration and ground-water flow are largely unobservable), and temporal changes (past and future) in boundary conditions.

The basic quantitative relations of physical hydrology are derived from fundamental laws of classical physics, particularly those listed in table 1.1. Derivations of hydrologic relations begin with a statement of the appropriate fundamental law(s) in mathematical form, with boundary and initial conditions appropriate to the situation under study, and are carried out by using mathematical operations (algebra and calculus). This is the approach that we will usually follow in the discussions of hydrologic processes in this text.

## 1.4 Dimensions and Units

### 1.4.1 Dimensions

Quantities determined by *measuring* take on a value corresponding to a point on the real number scale that is the ratio of the magnitude of the quantity to the magnitude of a standard unit of measure-

**Table 1.1** Summary of Basic Laws of Classical Physics Most Often Applied in Hydrologic Analyses.

#### Conservation of Mass

Mass is neither created nor destroyed.

#### Newton's Laws of Motion

1. The momentum of a body remains constant unless a net force acts upon the body (= conservation of momentum).
2. The rate of change of momentum of a body is proportional to the net force acting on the body, and is in the same direction as the net force (force equals mass times acceleration).
3. For every net force acting on a body, there is a corresponding force of the same magnitude exerted by the body in the opposite direction.

#### Laws of Thermodynamics

1. Energy is neither created nor destroyed (= conservation of energy).
2. No process is possible in which the sole result is the absorption of heat and its complete conversion into work.

#### Fick's First Law of Diffusion

A diffusing substance moves from where its concentration is larger to where its concentration is smaller at a rate that is proportional to the spatial gradient of concentration.

ment;<sup>1</sup> their dimensional quality is expressed in terms of the fundamental physical **dimensions** force [F] or mass [M], length [L], time [T], and temperature [Θ].

The fundamental dimensional character of measured quantities can be expressed as  $[M^a L^b T^c \Theta^d]$  or  $[F^e L^f T^g \Theta^h]$ , where the exponents  $a, b, \dots, h$  are integers or ratios of integers.

The choice of whether to use force or mass is a matter of convenience. Dimensions expressed in one system are converted to the other system via Newton's second law of motion:

$$[F] = [M L T^{-2}]; \quad (1.1a)$$

$$[M] = [F L^{-1} T^2]. \quad (1.1b)$$

The dimensions of energy are [F L] or  $[M L^2 T^{-2}]$ .<sup>2</sup> Some physical relations will be clearer if we use [E] to designate the dimensions of energy; thus we define

$$[E] \equiv [M L^2 T^{-2}] = [F L]. \quad (1.2)$$

Quantities obtained by *counting*, or as the ratio of measurable quantities with identical dimensions, are **dimensionless**; their dimensional character is denoted as [1].

---

Quantities obtained as logarithmic, exponential, and trigonometric functions are also dimensionless.<sup>3</sup>

Table A.2 gives the dimensional character of quantities commonly encountered in hydrology. Those with dimensions involving length only are classed as **geometric** (angle is included here also), those involving length and time or time only are **kinematic**, those involving mass or force are **dynamic**, and those involving temperature are **thermal** (latent heat is included here also).

### 1.4.2 Units

**Units** are the arbitrary standards in which the magnitudes of quantities are expressed. When we give the units of a quantity, we are expressing the ratio of its magnitude to the magnitude of an arbitrary standard with the same fundamental dimension (except, as noted, in the common temperature scales, where an additive term is also involved).

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The **Système International** (SI) is the international standard for all branches of science; it will be used throughout this text.

---

Hydrologists also encounter the **centimeter-gram-second (cgs) system**, which was an earlier version of the SI system. The **British**, or **common, system** is still the official measurement system of the United States, and so appears in reports of government agencies such as the US National Weather Service (NWS) and the US Geological Survey (USGS).

Largely because of the United States' retention of the British system, hydrologists commonly find it necessary to convert from one set of units to another; rules for doing this are given in appendix A.

---

It's important to observe unit conversion rules carefully to avoid egregious and embarrassing mistakes!

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### 1.4.3 Dimensional Properties of Equations

The two most important rules to incorporate into your thinking are:

1. An equation that completely and correctly describes a physical relation has the same dimensions on both sides of the equal sign, i.e., it is **dimensionally homogeneous**.
2. In equations, the dimensions and units of quantities are subject to the same mathematical operations as the numerical magnitudes.

A corollary of this latter rule is that only quantities with identical dimensional quality can be added or subtracted.

While there are no exceptions to the requirement of dimensional homogeneity, there are some important qualifications:

- Dimensional homogeneity is a *necessary* but not a *sufficient* requirement for correctly and completely describing a physical relation.
- Equations that are not dimensionally homogeneous can be very useful approximations of physical relationships.

This latter situation arises because the magnitudes of hydrologic quantities are commonly determined by the complex interaction of many factors, and it is often virtually impossible to formulate the physically correct equation or to measure all the relevant independent variables. Thus, hydrologists are often forced to develop and rely on relatively simple **empirical equations** (i.e., equations based on observed relations between measured quantities) that may be dimensionally inhomogeneous. Often, such equations are developed via the statistical process of **regression analysis**. Finally, it is important to recognize that

---

Equations can be dimensionally homogeneous but not unitarily homogeneous. (However, all unitarily homogeneous equations are of course dimensionally homogeneous.)

---

This situation can arise because each system of units includes "superfluous" units, such as miles (= 5,280 ft), kilometers (= 1,000 m), acres (= 43,560 ft<sup>2</sup>), hectares (= 10<sup>4</sup> m<sup>2</sup>), liters (= 10<sup>-3</sup> m<sup>3</sup>), etc.

As noted, dimensionally and/or unitarily inhomogeneous empirical equations are frequently encountered in hydrology. Because of this:

- The practicing hydrologist should check every equation for dimensional and unitary homogeneity.
- The units of each variable in an inhomogeneous equation **MUST** be specified.
- If you want to change the units used in an inhomogeneous equation, at least one of the coefficients or constants must change.

The above rules are crucial because

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If you use an inhomogeneous equation with units other than those for which it was given, you will get the wrong answer.

---

Surprisingly, it is not uncommon in the earth sciences and engineering literature to encounter inhomogeneous equations for which units are not specified—so *caveat calculator!*

In practice, there are often situations in which we want to use an inhomogeneous equation with quantities measured in units different from those used in developing it. The steps for determining the new numerical values when an inhomogeneous equation is to be used with new units are detailed in appendix A.

## 1.5 Properties of Water

Forces acting on water cause it to move through the hydrologic cycle, and the physical properties of water determine the qualitative and quantitative relations between those forces and the resulting motion. These physical properties are in turn determined by its atomic and molecular structures. Thus, although the detailed study of these structures and properties is outside the traditional scope of hydrology, it is important for the student of hydrology to have some understanding of them.

The physical properties of water are highly anomalous. As explained in more detail in appendix B,

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Most of the unusual properties of water are due to its being made up of polar molecules that form **hydrogen bonds** between adjacent water molecules and between water molecules and earth materials.

---

Here we briefly describe the properties of water most important to its behavior in the hydrologic cycle. These are summarized in table 1.2 and described in more detail in appendix B.

**Table 1.2** Summary of Properties of Liquid Water (see appendix B for more details).

Property	Uniqueness	Value at Surface	Importance
Melting and boiling points	Anomalously high for molecular weight.	Melting: 0°C Boiling: 100°C	Permits liquid water, as well as vapor and ice, to exist on earth's surface.
Density ( $\rho_w$ )	Maximum at 3.98°C, not at freezing point. Expands on freezing.	999.73 kg/m <sup>3</sup> (10°C)	Controls velocities of water flows. Lake and rivers freeze from top down; causes stratification in lakes.
Surface tension ( $\sigma$ )	Higher than most liquids.	0.074201 N/m (10°C)	Controls cloud droplet formation and raindrop growth; controls water absorption and retention in soils.
Viscosity ( $\mu$ )	Lower than most common liquids.	0.001307 N · s/m <sup>2</sup> (10°C)	Controls flow rates in porous media; low value results in turbulence in most surface flows.
Latent heat of vaporization ( $\lambda_v$ )	One of the highest known.	2.471 MJ/kg (10°C)	Controls land-atmosphere heat transfer and atmospheric circulation and precipitation.
Latent heat of fusion ( $\lambda_f$ )	Higher than most common liquids.	3,340 J/kg (0°C)	Controls formation and melting of ice and snow.
Specific heat (heat capacity) ( $c_w$ )	Highest of any liquid except ammonia.	4,191 J/kg · K (10°C)	Moderates air and water temperatures; determines heat transfer by oceans.
Solvent capacity	Excellent solvent for ionic salts and polar molecules.		Solution initiates erosion and transports erosion products; plant nutrients and CO <sub>2</sub> delivered in solution.

### 1.5.1 Freezing and Melting Temperatures

The hydrogen bonds that attract one water molecule to another can only be loosened (as in melting) or broken (as in evaporation) when the vibratory energy of the molecules is large—that is, when the temperature is high. Because of its anomalously high melting (273.16 K) and boiling temperatures (373.16 K), water is one of the very few substances that exists in all three physical states—solid, liquid, and gas—at earth-surface temperatures (figure 1.6). The Kelvin temperature unit and the Celsius temperature scale are defined by the freezing and melting temperatures of water.

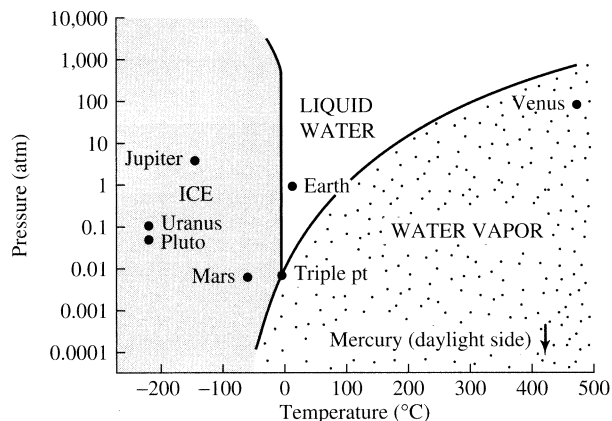
### 1.5.2 Density

**Mass density**,  $\rho_w$ , is the mass per unit volume [ $M L^{-3}$ ] of water, while **weight density** (or **specific weight**),  $\gamma_w$ , is the weight per unit volume [ $F L^{-3}$ ]. These are related by Newton's second law (i.e., force equals mass times acceleration):

$$\gamma_w = \rho_w \cdot g, \quad (1.3)$$

where  $g$  is the acceleration due to gravity [ $L T^{-2}$ ] ( $g = 9.81 \text{ m/s}^2$  at the earth's surface). Liquid water flows in response to spatial gradients of gravitational force and pressure (i.e., weight per unit area), so either  $\rho_w$  or  $\gamma_w$  appears in most equations describing the movement of liquid water.

The change in density of water with temperature is highly unusual (figure 1.7): liquid water at the freezing point is approximately 10% denser than ice and, as liquid water is warmed from  $0^\circ\text{C}$ , its density initially increases. This anomalous increase continues until density reaches a maximum of  $1,000 \text{ kg/m}^3$  at  $3.98^\circ\text{C}$ ; above this point the density decreases with temperature, as in most other substances.



In the SI system of units, the kilogram (kg) is defined as the mass of  $1 \text{ m}^3$  of pure water at its temperature of maximum density, and the newton (N) is the force required to impart an acceleration of  $1 \text{ m/s}^2$  to a mass of 1 kg (i.e.,  $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$ ). Note that the kilogram is commonly used as a unit of force as well as of mass: 1 kg of force ( $\text{kg}_f$ ) is the weight of a mass of 1 kg at the earth's surface. Thus, from equation (1.3),  $1 \text{ kg}_f = 9.81 \text{ N}$ .

The anomalous density behavior of water is environmentally significant. Because ice is less dense than liquid water, rivers and lakes freeze from the surface downward rather than from the bottom up. And, in lakes where temperatures reach  $3.98^\circ\text{C}$ , the density maximum controls the vertical distribution of temperature and causes an annual or semiannual overturn of water that has a major influence on biological and physical processes. However, except in modeling lake behavior,

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The variation of water density with temperature is small enough relative to measurement uncertainties that it can be neglected in most hydrological calculations.

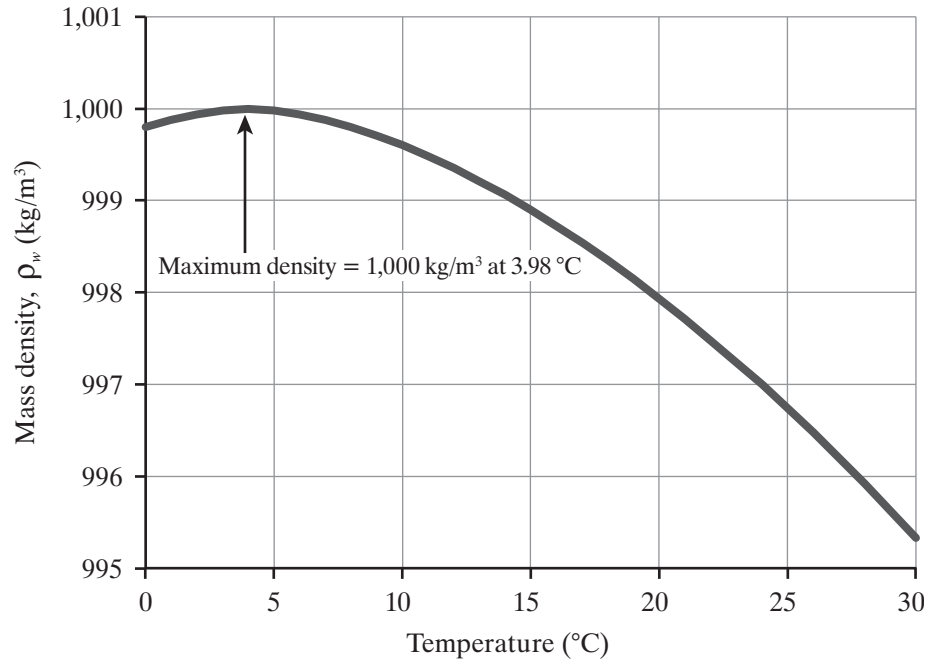
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### 1.5.3 Surface Tension

Molecules in the surface of liquid water are subjected to a net inward force due to hydrogen bonding with the molecules below the surface (figure 1.8). **Surface tension** is equal to the magnitude of that force divided by the distance over which it acts; thus its dimensions are [ $F L^{-1}$ ]. Surface tension can also be viewed as the work required to overcome that inward pull and increase the surface area of a liquid by a unit amount ( $[F L]/[L^2] = [F L^{-1}]$ ).

**Figure 1.6** Surface temperatures and pressures of the planets plotted on the phase diagram for water (Eagleson et al., *Opportunities in the Hydrologic Sciences* © 1991 by the US National Academy of Sciences. Reprinted with permission of the National Academy Press).

**Figure 1.7** Variation of density with temperature for pure water. It is highly unusual that the maximum density occurs at 3.98°C rather than at the freezing point (0°C), as it does for most liquids. It is also unusual that the solid form, ice, has a lower density (917 kg/m<sup>3</sup>) than the liquid at the freezing point. These anomalies have major impacts on the temperature structure of lakes, the behavior of rivers during freezing and thawing, the weathering of rocks, and other phenomena.



Surface tension significantly influences fluid motion where a water surface is present and where the flow scale is less than a few millimeters—i.e., in soils that are partially saturated or in which there is an interface between water and an immiscible liquid (e.g., hydrocarbons). As described in section 7.4.1, surface tension produces the phenomenon of **capillarity**, which affects soil-water distribution by pulling water into dry soils and holding soil water against the pull of gravity.

As might be expected from its strong intermolecular forces, water has a surface tension higher than most other liquids. Surface tension decreases rapidly as temperature increases, and this effect can be important when considering the movement of water in soils (see chapter 7). Dissolved substances can also increase or decrease surface tension, and certain organic compounds have a major effect on its value.

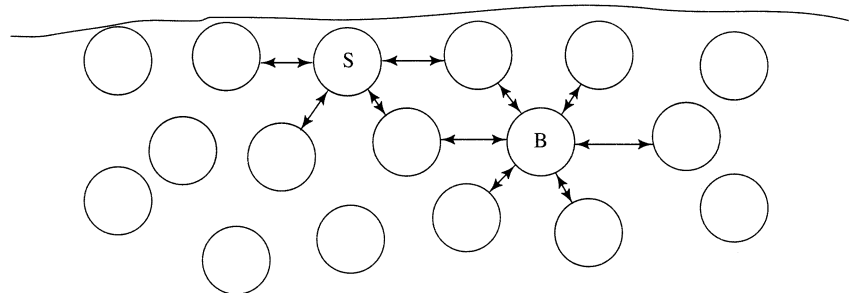
The relative importance of surface-tension force relative to gravitational force in water flows is quantitatively reflected in the dimensionless **Bond number**,  $Bo$ , given by

$$Bo \equiv \frac{\gamma_w \cdot L^2}{\sigma}, \quad (1.4)$$

where  $\sigma$  is surface tension,  $\gamma_w$  is weight density, and  $L$  is a characteristic length of the flow (e.g., soil-pore diameter or flow depth). In flows with  $Bo < 1$ , surface-tension forces exceed gravitational forces.

### 1.5.4 Viscosity and Turbulence

Flows of liquid water occur in response to gradients in gravity and/or pressure forces. **Viscosity** is the internal intermolecular friction that resists mo-



**Figure 1.8** Intermolecular forces acting on typical surface (S) and nonsurface (B) molecules.



tion of a fluid. An important concomitant of viscosity is the **no-slip condition**: the flow velocity at a stationary boundary is always zero, so that any flow near a boundary experiences a velocity gradient perpendicular to the boundary. At small spatial scales (centimeter scale or less) and low flow velocities (less than a few cm/s), viscous resistance controls the gradient and the rate of flow.

However—and somewhat surprisingly—the viscosity of water is low compared to other fluids because of the rapidity with which the intermolecular hydrogen bonds break and reform (about once every  $10^{-12}$  s). Thus, as flow scales and velocities increase, inertial effects soon dominate the effects of viscosity, so that formerly straight or smoothly curving flow paths become increasingly chaotic due to eddies. This phenomenon, called **turbulence**, produces a resistance to flow that depends on the flow scale and velocity, and is typically orders of magnitude larger than that due to viscosity. Hence, the physical relations describing subsurface flow in soil pores, where viscosity usually dominates, and in surface flows, where turbulence dominates, are very different.

The relative importance of turbulent and viscous resistance in a flow is quantitatively reflected in the dimensionless **Reynolds number**,  $Re$ :

$$Re \equiv \frac{U \cdot L \cdot \rho_w}{\mu} \quad (1.5)$$

where  $\rho_w$  is mass density,  $\mu$  is dynamic viscosity, and  $U$  is average velocity. In subsurface flows,  $L$  is the soil-pore diameter and flows with  $Re < 1$  are dominated by viscous resistance; in open-channel flows,  $L$  is the flow depth and flows with  $Re < 500$  are dominated by viscous resistance.

### 1.5.5 Latent Heats

Latent heat is energy that is released or absorbed when a given mass of substance undergoes a change of phase. Its dimensions are energy per mass,  $[E M^{-1}]$ , or  $[L^2 T^{-2}]$ . The term “latent” is used because no temperature change is associated with the gain or loss of heat. The large amounts of energy required to break hydrogen bonds during melting and vaporization, and which are released by the formation of bonds during freezing and condensation, make water’s latent heats very large relative to other substances.

The **latent heat of fusion** is the quantity of heat energy that is added or released when a unit mass of substance melts or freezes. For water, this is a signifi-

cant quantity, 3.34 KJ/kg. Latent heat of fusion plays an important role in the dynamics of freezing and thawing of water bodies and of water in the soil: Once the temperature is raised or lowered to  $0^\circ\text{C}$ , this heat must be conducted to or from the melting/freezing site in order to sustain the melting or freezing process.

The **latent heat of vaporization** is the quantity of heat energy that is added or released when a unit mass of substance vaporizes or condenses. Vaporization involves the complete breakage of hydrogen bonds, and water has one of the largest latent heats of vaporization of any substance. The latent heat of vaporization decreases with temperature. At  $10^\circ\text{C}$  its value is 2.471 MJ/kg, more than six times the latent heat of fusion and more than five times the amount of energy it takes to warm water from the melting point to the boiling point.

As discussed in chapters 2 and 3, water’s enormous latent heat of vaporization plays an important role in global heat transport (1) as a source of energy that drives the precipitation-forming process and (2) as a mechanism for transferring large amounts of heat from the earth’s surface into the atmosphere.

### 1.5.6 Specific Heat (Heat Capacity)

**Specific heat** (or **heat capacity**),  $c_w$ , is the property that relates a temperature change of a substance to a change in its heat-energy content. It is defined as the amount of heat energy absorbed or released per unit mass per unit change in temperature. Thus its dimensions are  $[E M^{-1} \Theta^{-1}] = [L^2 T^{-2} \Theta^{-1}]$ . The thermal capacity of water at  $10^\circ\text{C}$  is very high (4.191 KJ/kg K) and decreases slowly as temperature increases.

The temperature of a substance reflects the vibratory energy of its molecules. The heat capacity of water is very high relative to that of most other substances because, when heat energy is added to it, much of the energy is used to break hydrogen bonds rather than to increase the rate of molecular vibrations. This high specific heat has a profound influence on organisms and the global environment: It makes it possible for warm-blooded organisms to regulate their temperatures, and makes the oceans and other bodies of water moderators of the rates and magnitudes of ambient temperature changes.

### 1.5.7 Solvent Power

Because of the unique polar structure of water molecules and the existence of hydrogen bonds, almost every substance is soluble in water to some degree.

Ionic salts, such as sodium chloride, readily form ions that are maintained in solution because the positive and negative ends of the water molecules attach to the oppositely charged ions. Each ion is thus surrounded by a cloud of water molecules that prevents the ions from recombining. Other substances, particularly polar organic compounds such as sugars, alcohols, and amino acids, are soluble because the molecules form hydrogen bonds with the water molecules.

The importance of the solvent power of water to biogeochemical processes cannot be overstated. The first steps in the process of erosion involve the dissolution and aqueous alteration of minerals, and a significant portion of all the material transported by rivers from land to oceans is carried in solution (chapter 2). Virtually all life processes take place in water and depend on the delivery of nutrients and the removal of wastes in solution. In plants, the carbon dioxide necessary for photosynthesis enters in dissolved form (chapter 6); in animals the transport and exchange of oxygen and carbon dioxide essential for metabolism take place in solution.

## 1.6 Hydrologic Systems and the Conservation Equations

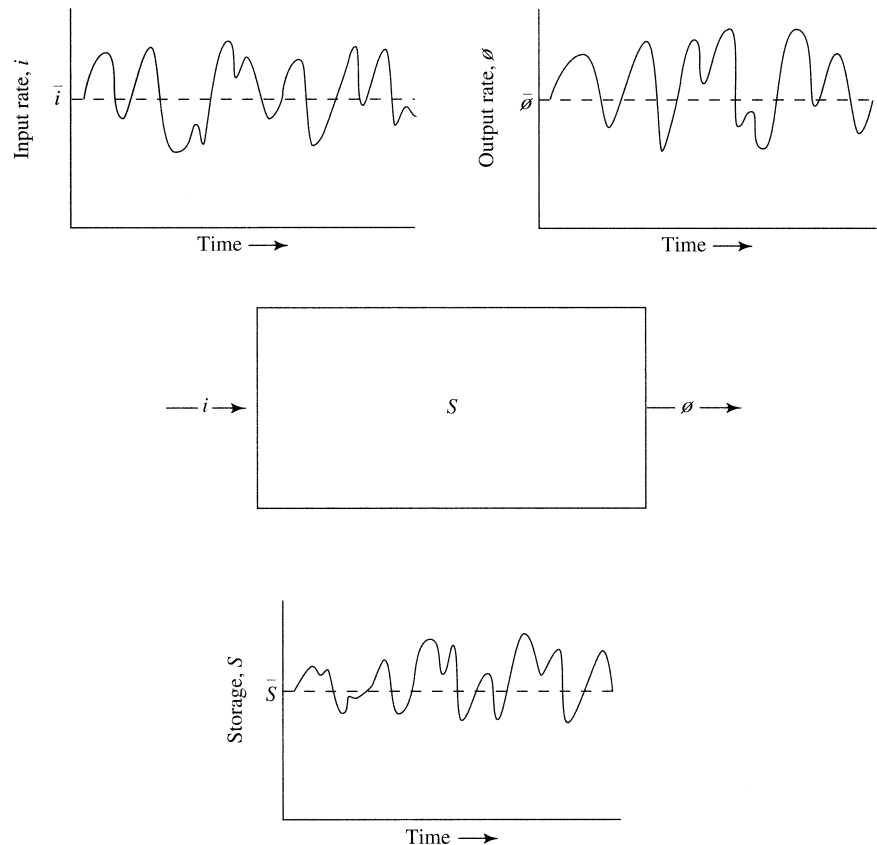
### 1.6.1 Hydrologic Systems

Several basic hydrologic concepts are related to the simple model of a **system** (as shown in figure 1.9).

- A system consists of one or more **control volumes** that can receive, store, and discharge a conservative substance.
- A **conservative substance** is one that cannot be created or destroyed within the system. These are **mass** ( $[M]$  or  $[F L^{-1} T^2]$ ), **momentum** ( $[M L T^{-1}]$  or  $[F T]$ ), and **energy** ( $[M L^2 T^{-2}]$  or  $[F L]$ ).

In most hydrologic analyses it is reasonable to assume that the mass density (mass per unit volume  $[M L^{-3}]$ ) of water is effectively constant; in these cases volume  $[L^3]$  (i.e., mass/mass density,  $[M]/[M L^{-3}]$ ) may be treated as a conservative quantity.

A control volume can be any conceptually defined region of space, and can be defined to include



**Figure 1.9** Conceptual diagram of a system.

regions that are not physically contiguous (e.g., the world's glaciers). Horton (1931, p. 192) characterized the range of scales of hydrologic control volumes:

Any natural exposed surface may be considered as a [control volume] on which the hydrologic cycle operates. This includes, for example, an isolated tree, even a single leaf or twig of a growing plant, the roof of a building, the drainage basin of a river-system or any of its tributaries, an undrained glacial depression, a swamp, a glacier, a polar ice-cap, a group of sand dunes, a desert playa, a lake, an ocean, or the earth as a whole.

The storages in figures 1.2 and 1.3 are systems linked by flows. The outer dashed line in figure 1.3 indicates that any group of linked systems can be aggregated into a larger system; the smaller systems could then be called **subsystems**.

### 1.6.2 The Conservation Equations

The basic conservation equation can be stated in words as:

The amount of a conservative quantity entering a control volume during a defined time period, minus the amount of the quantity leaving the volume during the time period, equals the change in the amount of the quantity stored in the volume during the time period.<sup>4</sup>

Thus the basic conservation equation is a generalization of (1) the conservation of mass, (2) Newton's first law of motion (when applied to momentum), and (3) the first law of thermodynamics (when applied to energy) (table 1.1). In condensed form, we can state the general conservation equation as

$$\text{Amount In} - \text{Amount Out} = \text{Change In Storage}, \quad (1.6)$$

but we must remember that the equation is true *only*: (1) for conservative substances; (2) for a defined control volume; and (3) for a defined time period.

If we designate the amount of a conservative quantity entering a region in a time period,  $\Delta t$ , by  $I$ , the amount leaving during that period by  $\emptyset$ , and the change in storage over that period as  $\Delta S$ , we can write equation (1.6) as

$$I - \emptyset = \Delta S. \quad (1.7)$$

Another useful form of the basic conservation equation can then be derived by dividing each of the terms in equation (1.7) by  $\Delta t$ :

$$\frac{I}{\Delta t} - \frac{\emptyset}{\Delta t} = \frac{\Delta S}{\Delta t}. \quad (1.8)$$

If we now define the average rates of inflow,  $\mu_I$ , and outflow,  $\mu_\emptyset$ , for the period  $\Delta t$  as follows:

$$\mu_I \equiv \frac{I}{\Delta t}, \quad (1.9)$$

$$\mu_\emptyset \equiv \frac{\emptyset}{\Delta t}, \quad (1.10)$$

we can write equation (1.8) as

$$\mu_I - \mu_\emptyset = \frac{\Delta S}{\Delta t}. \quad (1.11)$$

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Equation (1.11) states that the average rate of inflow minus the average rate of outflow equals the average rate of change of storage.

---

Another version of the conservation equation can be developed by defining the instantaneous rates of inflow,  $i$ , and outflow,  $\emptyset$ , as

$$i \equiv \lim_{\Delta t \rightarrow 0} \frac{I}{\Delta t}, \quad (1.12)$$

$$\emptyset \equiv \lim_{\Delta t \rightarrow 0} \frac{\emptyset}{\Delta t}. \quad (1.13)$$

Substituting these into equation (1.8) allows us to write

$$i - \emptyset = \frac{dS}{dt}. \quad (1.14)$$

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Equation (1.14) states that the instantaneous rate of input minus the instantaneous rate of output equals the instantaneous rate of change of storage.

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All three forms of the conservation equation, equations (1.7), (1.11), and (1.14), are applied in various contexts throughout this text. They are called **water-balance equations** when applied to the mass of water moving through various portions of the hydrologic cycle; control volumes in these applications range in size from infinitesimal to global and time intervals range from infinitesimal to annual or longer (figure 1.4). A special application of these equations, the regional water balance, is discussed in section 1.8, and an application of them to develop a model of watershed functioning is presented in

section 1.12. As indicated in figure 1.3, energy fluxes are directly involved in evaporation and snowmelt, and the application of the conservation equation in the form of **energy-balance equations** is essential to the understanding of those processes developed in chapters 5 and 6. Consideration of the conservation of momentum is important in the analysis of fluid flow, and this principle is applied in the discussion of turbulent exchange of heat and water vapor between the surface and the atmosphere in chapter 3.

## 1.7 The Watershed

### 1.7.1 Definition

Hydrologists commonly apply the conservation equation in the form of a water-balance equation to a geographical region in order to establish the basic hydrologic characteristics of the region. Most commonly, the region is a *watershed*:

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A **watershed** (also called **drainage basin**, **river basin**, or **catchment**) is the area that *topographically* appears to contribute all the water that passes through a specified cross section of a stream (the **outlet**) (figure 1.10). The surface trace of the boundary that delimits a watershed is called a **divide**. The horizontal projection of the area of a watershed is called the **drainage area** of the stream at (or above) the outlet.

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The watershed concept is of fundamental importance because it can usually be assumed that at least most of the water passing through the stream cross section at the watershed outlet originates as precipitation on the watershed, and the characteristics of the watershed control the paths and rates of movement of water as it moves over or under the surface to the stream network. To the extent this is true,

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Watershed geology, soils, topography, and land use determine the magnitude, timing, and quality of streamflow and ground-water outflow.

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Thus, the watershed can be viewed as a natural landscape unit, integrated by water flowing through the

land phase of the hydrologic cycle and, as William Morris Davis (1899, p. 495) stated,

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“One may fairly extend the ‘river’ all over its [watershed] and up to its very divides. Ordinarily treated, the river is like the veins of a leaf; broadly viewed, it is like the entire leaf.”

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Although political boundaries do not generally follow watershed boundaries, water-resource and land-use planning agencies recognize that effective management of water quality and quantity requires a watershed perspective. At the same time, it must be recognized that there are places in which topographically defined watershed divides do not coincide with the boundaries of ground-water flow systems; this is especially likely to occur in arid regions where topography is subdued and underlain by highly porous materials (e.g., Saudi Arabia, portions of the US Great Plains). This is discussed further in section 1.8.2.3.

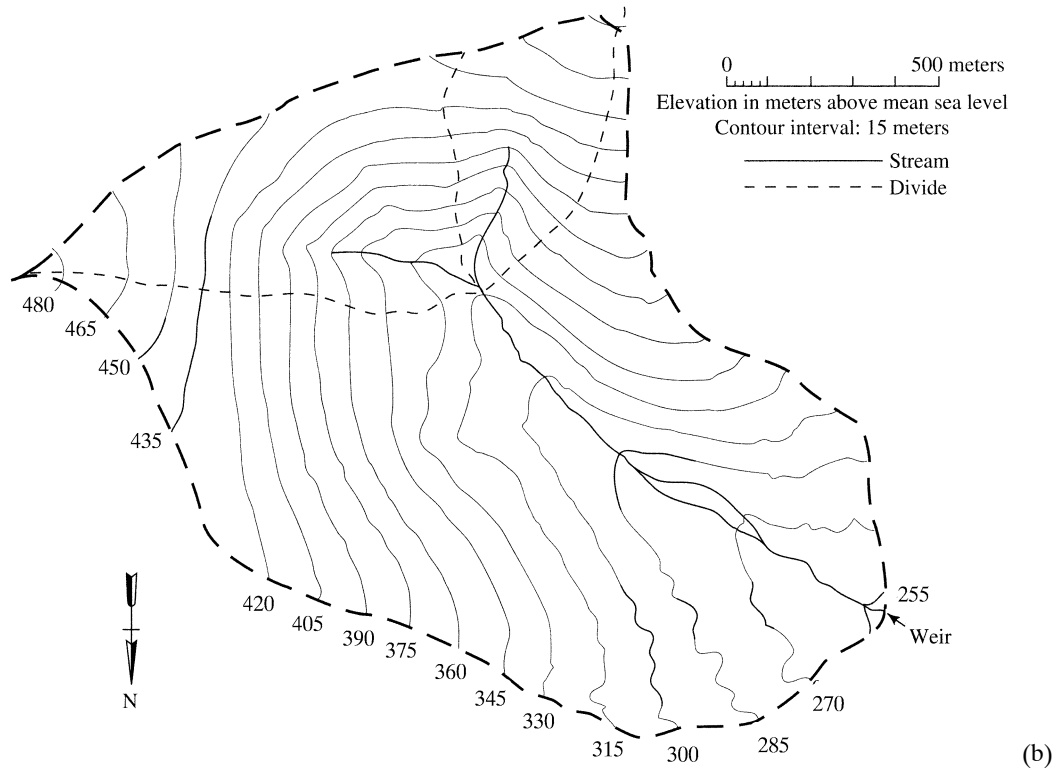
### 1.7.2 Delineation

Watershed delineation begins with selection of the watershed outlet: the location of the stream cross section that defines the watershed. This location is determined by the purpose of the analysis. For quantitative studies of water budgets or stream response, the outlet is usually a stream-gauging station where streamflow is continuously monitored. For geomorphic analyses of landscapes and stream networks, the outlets are usually at stream junctions or where a stream enters a lake or an ocean. For various water-resource analyses the outlet may be at a hydroelectric plant, a reservoir, a waste-discharge site, or a location where flood damage is of concern. As indicated in figure 1.10, upstream watersheds are nested within, and are part of, downstream watersheds.

#### 1.7.2.1 Manual Delineation

Although largely superseded by digital methods (see section 1.7.2.2), understanding the process of manual delineation provides valuable insight into the watershed concept. Furthermore, digital watershed delineations often contain errors, so it is essential to check them.

Manual watershed delineation requires a topographic map (or stereoscopically viewed aerial photographs). To trace the divide, start at the location of the chosen watershed outlet, then draw a line away from the left or right stream bank, *maintaining the line*



**Figure 1.10** (a) Oblique aerial photograph of Glenn Creek Watershed, Fox, Alaska, looking southeast. Discharge-measurement weir is visible near center of photograph. (b) Glenn Creek Watershed and tributary watersheds delineated on a topographic map.

*perpendicular to the contour lines.* Frequent visual inspection of the contour pattern is required as the divide is traced out to assure that an imaginary drop of water falling streamward of the divide would, if the ground surface were imagined to be impermeable, flow downslope and eventually enter the stream network upstream of the outlet. Continue the line until its trend is generally opposite to the direction in which it began, and is generally above the headwaters of the stream network. Then return to the starting point and trace the divide from the other bank, eventually connecting with the first line.

Note that a divide can never cross a stream, though there are rare cases where a divide cuts through a wetland (or, even more rarely, a lake) that has two outlets draining into separate stream systems. The lowest point in a drainage basin is always the basin outlet, i.e., the starting point for the delineation. The highest point is usually, but not necessarily, on the divide.

### 1.7.2.2 Digital Delineation

In recent years there has been a rapid development of readily accessible and generally reliable digital tools for watershed delineation. These are based on **digital elevation models (DEMs)**, which are computer data files that give land-surface elevations at grid points. The DEM elevations are based on radar reflections collected by satellite. The original data usually contain many errors due to false readings from vegetation, areas of radar shadowing by topography, lack of reflections from water surfaces, and other effects. Thus elaborate techniques are required for removing spurious depressions and rises, filling in areas subject to shadowing, and incorporating previously digitized stream networks (Tarboton et al. 1991; Martz and Garbrecht 1992; Tarboton 1997; Verdin and Verdin 1999; Lehner et al. 2008; Pan et al. 2012). However, different techniques may provide widely differing results, as found by Khan et al. (2013) for the Upper Indus River Watershed in Pakistan.

Currently, there are two web-based services that provide automated watershed delineation. In the United States, the USGS provides the StreamStats (<http://water.usgs.gov/osw/streamstats>) application that not only delineates watersheds for user-selected basin outlets, but also provides data on a large number of watershed characteristics and measured or estimated streamflow statistics. Globally, a team of scientists connected with the World Wildlife Fund has developed the HydroSHEDS database ([\[hydrosheds.cr.usgs.gov\]\(http://hydrosheds.cr.usgs.gov\)\) describing the earth's topography, drainage networks, and watersheds at three resolutions: 90, 500, and 1,000 m. Figure 1.11 shows the HydroSHEDS map of the major watersheds of Africa.](http://</a></p>
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The automated approach to watershed delineation allows the concomitant rapid extraction of much hydrologically useful information on watershed characteristics (such as the distribution of elevation and land-surface slope) that could previously be obtained only by very tedious manual methods.

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## 1.8 The Regional Water Balance

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The regional water balance is the application of the conservation of mass equation to the water flowing through a watershed or any land area, such as a state or continent.

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The upper surface of the control volume for application of the conservation equation is the surface area of the watershed (or other land area); the sides of the volume extend vertically downward from the divide some indefinite distance assumed to reach below the level of significant ground-water movement.

In virtually all regional hydrologic analyses, it is reasonable to assume a constant density of water because its density changes little with temperature, and any variation is much smaller than the uncertainties in the measured quantities. Thus we can treat volume  $[L^3]$  rather than mass as a conservative quantity. For comparative analyses of hydrologic climate it is useful to divide the volumes of water by the surface area of the region, so that the quantities have the dimension  $[L]$  ( $= [L^3]/[L^2]$ ).

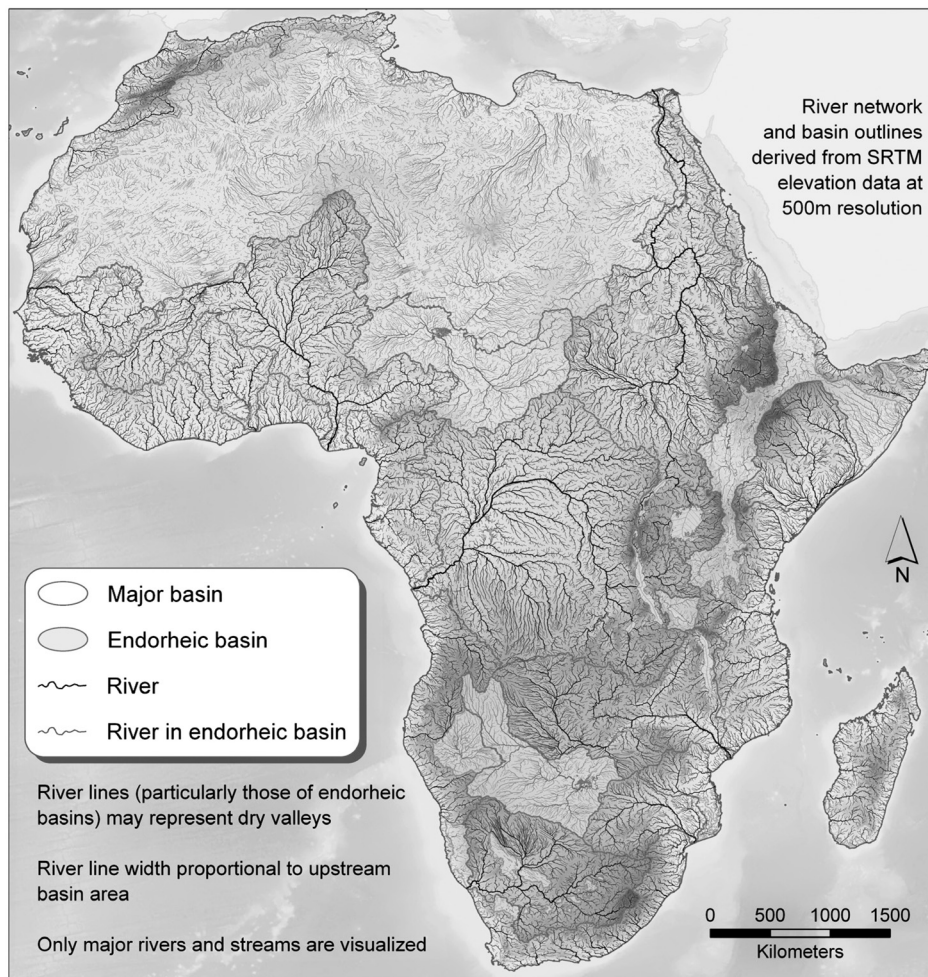
Computation of the regional water balance is a basic application of hydrologic concepts because

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Evaluation of the regional water balance provides the most basic characterization of a region's hydrology and potential water resources.

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In this section we will first develop a conceptual regional water balance, from which we can define some useful terms and show the importance of climate in determining regional water resources, following which we consider some of the observational challenges intrinsic to hydrology.



**Figure 1.11** HydroSHEDS map of major African watersheds and rivers [Lehner et al. (2008). New global hydrography derived from spaceborne elevation data. *Eos* 89(10):93–104, with permission of the American Geophysical Union].

### 1.8.1 The Water-Balance Equation

Consider the watershed shown in figure 1.12. For any time period of length  $\Delta t$  we can write the water-balance equation as

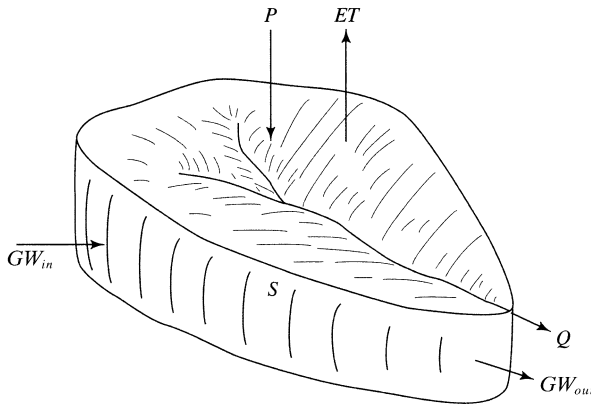
$$P + GW_{in} - (Q + ET + GW_{out}) = \Delta S, \quad (1.15)$$

where  $P$  is precipitation (liquid and solid),  $GW_{in}$  is ground-water inflow (liquid),  $Q$  is stream outflow (liquid),  $GW_{out}$  is ground-water outflow (liquid), and  $\Delta S$  is the change in all forms of storage (liquid and solid) over the time period.  $ET$  is **evapotranspiration**, the total of all water that leaves a region as vapor via direct evaporation from surface-water bodies, snow, and ice, plus **transpiration** (water evaporated after passing through the vascular systems of plants; the process is described in section 6.5). All the quantities in (1.15) are total amounts for the period  $\Delta t$ . If we

average the water-balance quantities over a reasonably long time period (say, many years), we can write the water balance as

$$\mu_P + \mu_{GW_{in}} - [\mu_Q + \mu_{ET} + \mu_{GW_{out}}] = \mu_{\Delta S}, \quad (1.16)$$

where  $\mu$  denotes the long-term average of the subscript quantity. To be representative of the hydrologic climate of a region, the time period used for averaging should be “many” years long, so that the annual climatic cycle and, hopefully, the major interannual variability is averaged out. Thus ideally  $\Delta t$  would be decades long—on the order of the 30-yr period used to compute climatic normal values in the United States. However, as one considers periods of many decades and longer, the possibility of significant trends due to climate or land-use changes increases and the validity of long-term averages becomes uncertain. This prob-



**Figure 1.12** Schematic three-dimensional diagram of a watershed, showing the components of the regional water balance:  $P$  = precipitation,  $ET$  = evapotranspiration,  $Q$  = stream outflow,  $GW_{in}$  = ground-water inflow,  $GW_{out}$  = ground-water outflow.

lem of *nonstationarity* is one of the major challenges to hydrology; it is discussed further in section 1.9.2.3.

Unless there are obvious reasons for thinking otherwise (e.g., melting of glaciers, construction of large reservoirs, large-scale water transfers in or out, extensive pumping and export of ground water), it is commonly assumed that water storage is not significantly increasing or decreasing over time, so that  $\mu_{\Delta S} \approx 0$ . And, because watersheds are topographically defined and ground-water flow is driven by gravity (section 8.2), we can often also assume that ground water and watershed divides coincide so that  $GW_{in}$  is negligible. With these assumptions, we can write (1.16) as

$$\mu_Q + \mu_{GW_{out}} = \mu_P - \mu_{ET}. \quad (1.17)$$

Note that the units of the quantities in (1.16) and (1.17) are *rates*; i.e., their dimensions are  $[L T^{-1}]$  or  $[L^3 T^{-1}]$ .

We now introduce an important definition:

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The sum of streamflow and ground-water outflow is called **runoff (RO)**,<sup>5</sup> i.e.,  

$$RO = Q + GW_{out}$$

---

Therefore,

$$\mu_{RO} = \mu_P - \mu_{ET}. \quad (1.18)$$

Runoff is the rate at which liquid water leaves the region, and

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The average runoff represents the maximum rate at which nature makes water available for human use and management, i.e., it is the **potential water resource** in a region.

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However, as we will explore later in this chapter,

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The temporal variability of runoff must be evaluated in assessing the *actual* regional water-resource availability.

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As we will see in chapter 6, evapotranspiration is determined largely by meteorologic variables, so both precipitation and evapotranspiration can be considered to be externally imposed climatic “boundary conditions.” Thus, from equation (1.18),

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Runoff is a residual or difference between two climatically determined quantities.

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One obvious implication of these results is that natural and anthropogenic climate changes will have impacts on water resources—a phenomenon that has recurred frequently in human history.

### 1.8.2 Evaluation of Water-Balance Components

Evaluation of the regional water balances obviously involves measurement of its components. In this section we introduce the major challenges involved in measuring the terms in equation (1.16) and in justifying the assumptions leading to equation (1.18). These challenges are some of the central focuses of current hydrologic research; they are discussed in more detail in later chapters addressing individual water-balance components.

Note that all components of the water balance are subject to interannual variability and perhaps cycles and trends as well as seasonal variability, so evaluation of the long-term average value of each component (even at a single measurement station) involves careful application of statistical principles, some of which are presented in appendix C. And, as in any discipline, every hydrologic measurement is subject to some uncertainty; the evaluation of this uncertainty is discussed in section 1.11.



### 1.8.2.1 Streamflow

In most humid regions, at least, streamflow is usually by far the major component of runoff. If the region of interest is a watershed, streamflow can be measured at a single point—a **gauging station** at the watershed outlet. Continuous monitoring at gauging stations is expensive because it requires elaborate on-site instrumentation and continued calibration (see appendix E), but it can usually be done relatively precisely. If the water balance is to be computed for a region other than a watershed (e.g., country or continent), spatial as well as temporal averages of measurements at several gauging stations must be computed. This presents a statistical challenge because of the typically uneven distribution of gauging stations, the nested nature of watersheds, the effects of scale, and other complications that increase uncertainty.

Despite the importance of streamflow data, maintenance of a comprehensive global river monitoring network faces numerous technological, economic, and institutional obstacles. Because of this, the number of gauging stations and access to river discharge information has been declining since the 1980s, especially in developing nations (Vörösmarty et al. 1999; International Association of Hydrological Sciences 2001). Even in developed countries, the streamflow-measuring network is frequently informationally sparse (Mishra and Coulibaly 2009). As a result, there has been intensive interest in developing satellite-based methods for measuring streamflow (Bjerklie et al. 2003, 2005; Bjerklie 2007; Smith and Pavelsky 2008; Durand et al. 2014). However, remote-sensing techniques are considerably less precise than on-site measurements.

### 1.8.2.2 Precipitation

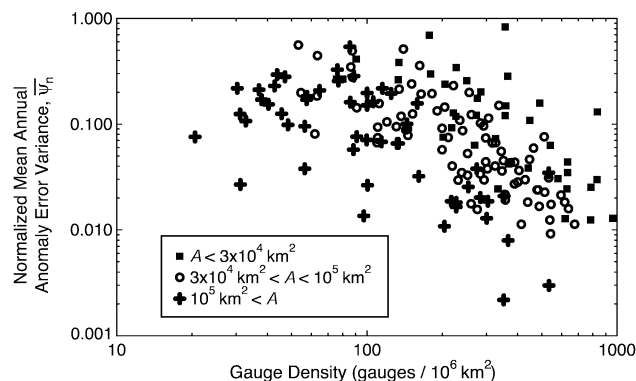
The measurement of precipitation at a point is conceptually simple but, as examined in chapter 4,

subject to many sources of error. The error is compounded when snow is involved (chapter 5). Computation of areal average values requires consideration of the density and distribution of individual gauges (most gauges are located at relatively low elevations in populated areas), the degree to which observations at nearby gauges are duplicative, and other factors.

Milly and Dunne (2002) examined sources of error in precipitation measurements as part of a global study of water balances in large watersheds (average area = 51,000 km<sup>2</sup>). They considered three additive sources of error: (1) poor distribution of gauges, (2) errors in measurements at individual gauges (section 4.2.1.2.7), and (3) failure to sample high elevations, where precipitation usually increases due to *orographic effects* (section 4.1.5). In watersheds with the highest precipitation errors, the main error source was failure to sample high elevations. Overall, however, the three error sources varied in importance. Figure 1.13 shows that there is only a very general decrease in error with gauge density, and that densities at least on the order of 100 gauges/10<sup>6</sup> km<sup>2</sup> are required to achieve errors in the range of 10% in assessing water balances in large watersheds. Although this density seems sparse (it is equivalent to one gauge in an area 100 km on a side), such densities are found only in a very small fraction of the global land area (Milly and Dunne 2002).

### 1.8.2.3 Ground-Water Inflow and Outflow

Regional ground-water inflow and outflow rates are not directly measurable. The best information one can hope for is monitoring of water-level elevations in strategically located wells along with detailed knowledge of the regional geology and in-situ measurement of the hydraulic conductivity of important geologic formations. If enough information and resources are avail-



**Figure 1.13** Scatter plot of relative error in watershed average precipitation as a function of gauge density. Errors are smaller for larger watersheds [Milly and Dunne (2002). Macroscale water fluxes: 1. Quantifying errors in the estimation of basin mean precipitation. *Water Resources Research* 38(10), with permission of the American Geophysical Union].

able, the magnitudes of the ground-water terms can be assessed by developing a ground-water model of the region. This information is seldom available, so the usual practice is to use existing geologic and topographic information to infer, at least qualitatively, the magnitudes of the ground-water terms. Very commonly, this information is not available, and these components are simply assumed to have negligible magnitude.

However, as explained in section 9.2, there are many topographic and geologic situations in which upstream watersheds contribute water to regional ground-water systems that ultimately appears in the streams draining downstream watersheds, and recent research suggests that ground-water flows are often significant components of the regional water balance. Schaller and Fan (2009) found that  $\mu_Q/\mu_{RO}$  for 1,555 watersheds in the United States ranged from 0.03 to 8.9, with half the watersheds being importers of ground water ( $\mu_Q/\mu_{RO} > 1$ ) and half being exporters ( $\mu_Q/\mu_{RO} < 1$ ). They concluded that the distribution of importing and exporting watersheds is well correlated with climatic trends, with higher values of  $\mu_Q/\mu_{RO}$  associated with more arid regions, and that climate may be used as a first-order predictor of  $\mu_Q/\mu_{RO}$ . Other influencing factors were (1) geology; (2) watershed size (larger watersheds tend to collect the ground-water flows of tributaries and have  $\mu_Q/\mu_{RO}$  closer to 1; and (3) relative position or elevation of a watershed, i.e., headwaters versus coastal zones. This is discussed further in section 9.5.5.2.3.

#### 1.8.2.4 Storage

The net change in watershed storage over a period of observation is the difference between the amount of water stored as ground water and in rivers, lakes, soil, vegetation, and snow and ice at the end of the measurement period and the amount stored at the beginning of the period. If  $S_t$  represents the watershed storage at the end of year  $t$  and  $\Delta S_t \equiv S_t - S_{t-1}$  is the change in storage over year  $t$ , then the average change in storage over a  $T$ -yr period,  $\mu_{\Delta S}$ , is

$$\mu_{\Delta S} = \frac{\sum_{t=0}^T \Delta S_t}{T} = \frac{S_T - S_0}{T}. \quad (1.19)$$

Until very recently, the only way of directly observing storage changes was to combine periodic representative observations of all storage components distributed over the region of interest. Such observations—particularly of ground water, which is usually the largest storage reservoir—are virtually never avail-

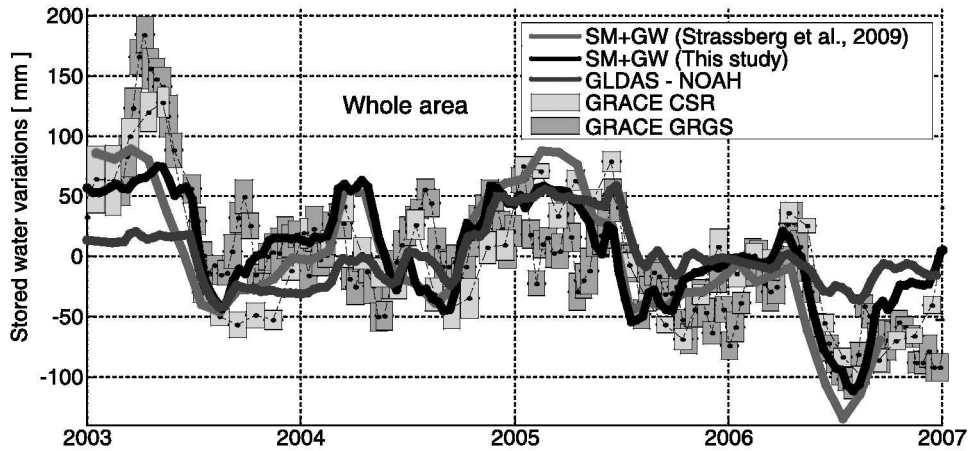
able. Lacking these, hydrologists attempt to minimize the value of  $\mu_{\Delta S}$  by (1) using long measurement periods (large  $T$ ) and (2) selecting the time of beginning and end of the measurement period such that storage values are likely to be nearly equal (small  $\Delta S_t$ ). In the United States, the USGS attempts to minimize the values of  $\Delta S_t$ , and hence  $S_T - S_0$ , by beginning the **water year** on 1 October, on the assumption that by this time transpiration will have largely ceased and soil moisture and ground-water storage will have been recharged to near their maximum levels. However, other water-year spans may be more appropriate for specific regions—for example, in northern areas the time of disappearance of the annual snowpack in the spring may be a time when watershed storage is likely to be full.

A new tool for directly observing regional storage changes became available in 2002, when the Gravity Recovery and Climate Experiment (GRACE) satellite mission was launched. The GRACE mission consists of two satellites at an altitude of  $\sim 450$  km in an identical polar orbit. Mass redistribution on earth is measured by precisely monitoring the distance between the two satellites and tracking their positions via global-positioning-system satellites (Tapley et al. 2004). The GRACE signal reflects changes in vertically integrated stored water, including snowpacks, glaciers, surface water, soil moisture, and ground water at all depths. The system can measure mass changes equivalent to  $\sim 1$  cm of water at the land surface over a distance of a few hundred kilometers or more, and the measurements can be used to assess total water-storage changes in regions over many hundreds to thousands of kilometers. At these scales, many studies have shown that GRACE-derived stored-water observations compare well with ground-based measurements and hydrological models (figure 1.14), and indicate that GRACE can be used to monitor hydrological systems and improve hydrological modeling (Lettenmaier and Famiglietti 2006; Güntner 2008; Ramillien et al. 2008; Longuevergne et al. 2010).

However, if the spatial scale of interest is less than a few hundred kilometers, or if the time period for averaging includes periods before 2002, GRACE data cannot be used. Thus, hydrologists often face a near-total absence of observations of storage changes and must revert to the older strategy—which usually amounts to simply assuming that  $\mu_{\Delta S}$  is negligible.

#### 1.8.2.5 Evapotranspiration

Like ground water, regional evapotranspiration rates are not directly measurable. Unlike ground wa-



**Figure 1.14** Comparison between GRACE-derived stored-water variations for the High Plains aquifer (United States) with storage estimates derived from ground-based measurements. GRACE estimates are shown as rectangles (height reflects uncertainty) using two different methodologies. Continuous curves are ground-based estimates using three different methodologies [Longuevergne et al. (2010). GRACE hydrological estimates for small basins: Evaluating processing approaches on the High Plains Aquifer, USA. *Water Resources Research* 46, with permission of the American Geophysical Union].

ter, however, evapotranspiration rate is virtually always a significant, and very often the major, component of the water balance. Thus, one of the major challenges to hydrologic science is the determination of regional evapotranspiration.

As discussed in detail in chapter 6, local evapotranspiration rate,  $et$ , is determined by solar radiation input,  $K$ ; net infrared radiation input,  $L$ ; air temperature,  $T_a$ ; relative humidity,  $RH$ ; wind speed,  $u_a$ ; soil-water content,  $\theta$ ; vegetation height,  $z_{veg}$ ; and leaf area,  $LAI$ :

$$et = f_{et}(K, L, T_a, RH, u_a, \theta, z_{veg}, LAI). \quad (1.20)$$

Thus computation of regional evapotranspiration rate requires representative regional information about a large number of meteorological, soil, and vegetation variables and a functional model for  $f_{ET}$ . However, even in an intensely instrumented research watershed, Kampf and Burges (2010) found that various forms of  $f_{ET}$  gave  $et$  estimates that varied by 18%.

Since one cannot expect to have reliable measurements of all the dependent variables in (1.20) over an extensive area, other strategies must be used to estimate  $et$ . Various approaches to this problem are described in section 6.8. However, because the direct computation of average regional evapotranspiration is so intractable, it is often estimated from the

regional water balance via equation (1.17), usually with the additional assumption that  $GW_{out}$  is negligible, i.e., as

$$\mu_{ET} = \mu_P - \mu_Q. \quad (1.21)$$

Equation (1.21) is probably the most common application of hydrologic analysis. For example, in a global analysis of the effects of vegetation density and type on evapotranspiration, Peel et al. (2010) used (1.21) to compute evapotranspiration for 861 globally distributed watersheds with ostensibly good measurements of  $\mu_P$  and  $\mu_Q$ . Interestingly, equation (1.21) gave *negative* values of  $\mu_{ET}$  for 114 (13%) of the watersheds. The possible reasons for this physically impossible result illustrate the kinds of measurement challenges that hydrologists face: (1) underestimation bias in the watershed average precipitation estimate; (2) unknown error in the streamflow observations; (3) unknown anthropogenic interwatershed water transfers; or (4) existence of significant unknown subsurface flows ( $GW_{in}$  or  $GW_{out}$ ). Peel et al. (2010) judged that the first of these was the most likely cause of error in this case. Note that the results of Schaller and Fan (2009) also indicate that the assumption of negligible ground-water inflows and outflows may often introduce errors into estimates of evapotranspiration made via equation (1.21).

### 1.8.3 Summary

The regional water balance is a concise statement of a region's hydrologic climate and available water resources. However, the above discussions make clear that evaluating the terms in the water-balance equation presents major conceptual and measurement challenges. Much of the rest of this text focuses on methods for dealing with these challenges. In particular, the next two sections of this chapter introduce some of the conceptual issues that confront the characterization of quantities that vary spatially and through time.

In addition to these scientific and technical issues, it is important to note that the information available for hydrologic studies is subject to social and political influences because

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Measurements of critical water-balance quantities are carried out by governments, and require continued commitments of resources.

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## 1.9 Special Characteristics of Hydrologic Variables

Statistical techniques were developed to make inferences about the properties of a target **population** from the properties of **samples** drawn from that population. In classical statistics, inferences are based on the assumptions that (1) every element of the sample is equally representative of the population, (2) every element of the sample has an equal chance of being selected, and (3) the confidence in inferences can be increased by increasing the sample size.

Because hydrologic variables are distributed in either space or time or both, one or more of these assumptions is often not satisfied. The sample values of spatially distributed quantities are influenced by geologic, topographic, vegetational, climatic, and cultural factors that produce regional trends, redundancies, discontinuities, and sampling gaps. Sample values distributed in time are subject to temporal trends, seasonal and other cycles, and unavoidable sampling limitations (i.e., one cannot go back in time to sample). The following subsections introduce some of the approaches that hydrologists use to deal with the challenges of spatially and temporally distributed variables. More detailed discussion of statistical concepts is given in appendix C, and in Helsel and Hirsch (1992).

### 1.9.1 Spatial Variability

Precipitation, soil moisture, evapotranspiration, ground-water levels, and the properties of topography, snowpack, soil, and vegetation that influence hydrologic processes vary spatially over the geographic regions that constitute control volumes for hydrologic analyses. Descriptions of spatial variability are based on measurements made at discrete points (e.g., precipitation gauges). These measurements, which may be expressed as time averages, constitute spatial samples.

Traditional statistical methods, such as those described in appendix C, can be used to compute spatial averages and measures of spatial variability from the point values. However, spatially distributed quantities usually have two characteristics that can cause conventional statistical measures to be misleading: (1) measurement points are unevenly distributed over the region and (2) values measured at “nearby” points are likely to be similar and, to some extent, redundant. The point values are therefore almost always an unrepresentative sample of the true field, and commonly large portions of the area have no information.

Because of this, and because of the importance of accurately quantifying variables such as precipitation, special techniques have been developed for characterizing and accounting for spatial variability. The essence of the most widely used of these techniques is to examine the measured point values to identify a mathematical model that relates the average difference  $\Delta p(d)$  in values observed at two points a distance  $d$  apart. Such a model can be represented as

$$\Delta p(d) \equiv E[p(x_i, y_i) - p(x_j, y_j)] = f_d(d) \quad (1.22)$$

where  $p(x_i, y_i)$  and  $p(x_j, y_j)$  are the observations at map coordinates  $(x_i, y_i)$  and  $(x_j, y_j)$ ,  $E[\dots]$  indicates the average of the expression in brackets,  $d$  is the distance between the two points, and  $f_d(d)$  is the mathematical model. Once a suitable model is identified, it can be used with the observed values to estimate values of  $p$  at grid points throughout the region of interest. These observed and estimated values can then be contoured, and the contours integrated to determine the regional average. Details of these techniques are discussed in the context of rainfall analysis in section 4.3; however, they apply to other spatially distributed quantities as well.

Increasingly, statistical concepts are being developed to facilitate combining ground-level measurements of precipitation with satellite or radar observa-

tions to enhance information about areal distribution (e.g., Anagnostou et al. 2010; AghaKouchak et al. 2011; Lin and Wang 2011; Xie and Xiong 2011).

## 1.9.2 Temporal Variability

### 1.9.2.1 Time Series

As noted, the inputs, storages, and outputs in figures 1.1, 1.2, and 1.8 are all time-distributed variables. Thus, the concept of time variability is inherent to the concept of the hydrologic system, and we have seen how time averaging is applied to develop alternative forms of the conservation equations. Descriptions and comparisons of time-distributed variables in terms of their average value, variability, and other characteristics are made by applying the statistical methods described in appendix C to data samples called *time series*:

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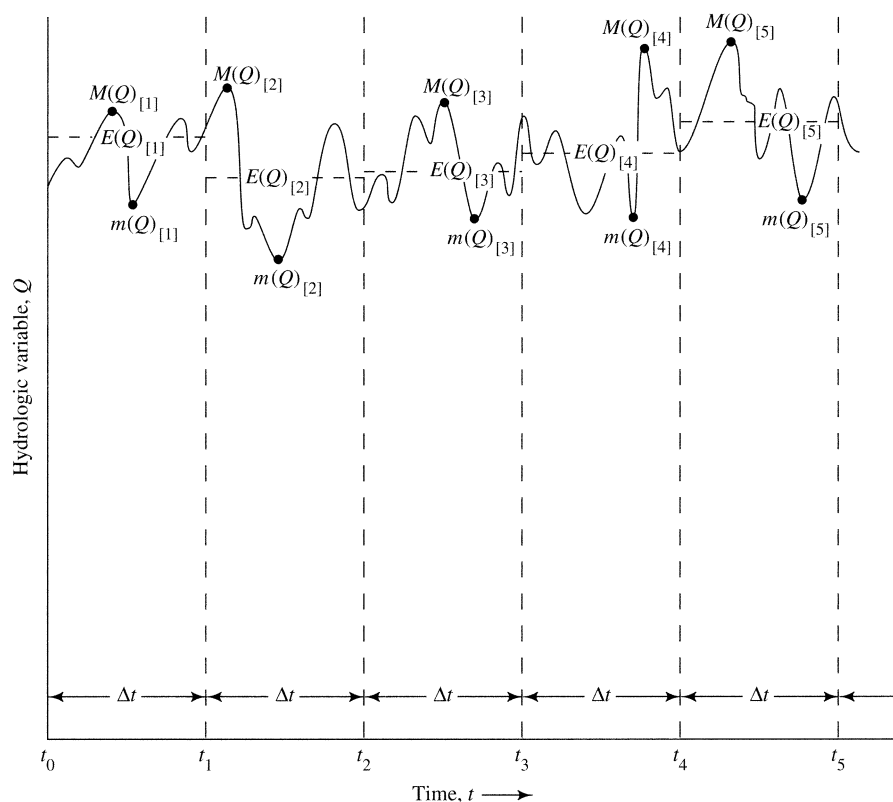
A **time series** is a time-ordered sequence of discrete values of a variable separated by a constant time interval  $\Delta t$ .

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Time series are conventionally treated as representative samples of the long-term behavior of the variable (but see the discussion in section 1.9.2.3).

For all time-series analyses, the time interval,  $\Delta t$ , must be selected and the total time period of interest divided into intervals of length  $\Delta t$ . Usually  $\Delta t = 1$  yr for water-balance or flood analysis, but for other purposes  $\Delta t$  might be 1 day or 1 month. Some hydrologic time-series variables are inherently **discrete**—for example, the number of days with more than 25 mm rain in each year at a particular location. However, variables like precipitation, evapotranspiration, and streamflow are **continuous** time traces that take on values at every instant in time.

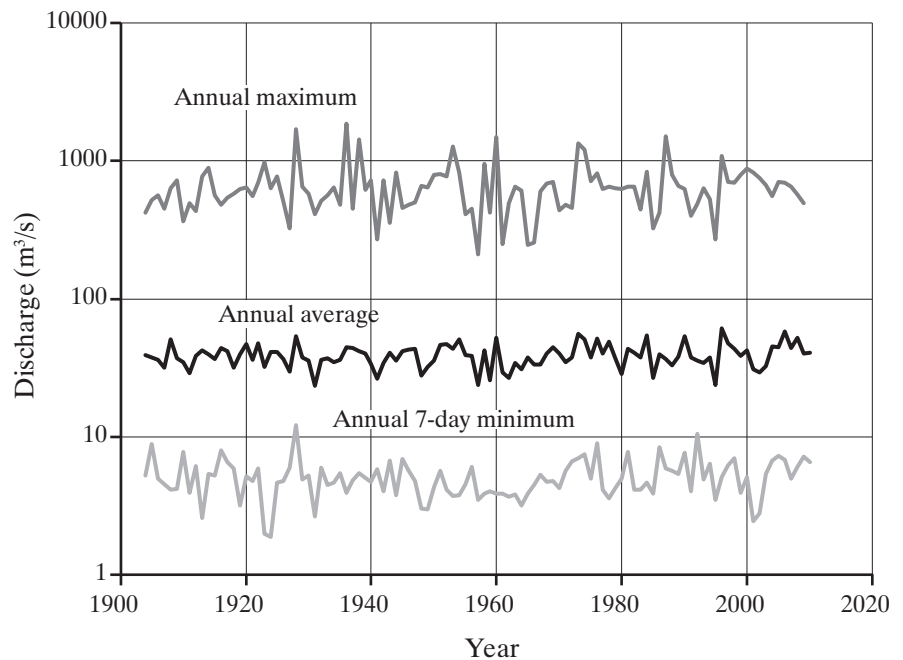
For statistical analysis, one must convert a continuous trace into discrete form. To do this, first specify the time interval, then, depending on the purpose of the analysis, select a single value of the variable of interest associated with each interval. Figure 1.15 and table 1.3 show typical values used to discretize time-series samples for various types of analyses, and figure 1.16 shows three time series developed from the continuous measurements at a streamflow-gauging station. In all three cases,  $\Delta t = 1$  yr, but different dis-



**Figure 1.15** Schemes for converting a continuous time trace into a discrete time series. For each  $\Delta t$ , one may select the average  $[E(Q)]$ , maximum  $[M(Q)]$ , or minimum  $[m(Q)]$ .

**Table 1.3** Examples of Time Series Discretization for Various Analyses.

Variable	Purpose of Analysis	Value Used for Discretization
precipitation, evapotranspiration	water balance	annual total or average
streamflow	flood	annual instantaneous maximum
streamflow	water balance	annual total or average
streamflow	drought, low flow	annual minimums of overlapping consecutive day ( $d$ ) averages; $d = 1, 7, \dots, 180$
storage	water balance	value at beginning or end of year



**Figure 1.16** Traces of the annual maximum daily streamflow, annual average daily streamflow, and annual minimum 7-day average streamflow for the Pemigewasset River, New Hampshire.

charge values are selected for different analyses. It is important to note that time series developed from a single continuous time trace by choosing different discretizing schemes and/or different  $\Delta t$  values will in general have very different statistical characteristics.

**1.9.2.2 Temporal Variability of Streamflow**

As noted in section 1.8.1, the long-term average streamflow rate,  $\mu_Q$ , is highly significant because it represents the rate at which water is potentially available for human use and management (assuming negligible ground-water outflow). However, due to the seasonal and interannual variability of precipitation, snowmelt, and evapotranspiration, streamflow is highly variable over time: even in humid regions unregulated streamflow at a given location typically varies over three or more orders of magnitude. Be-

cause of this large time variability, we cannot rely on the average flow to be available most of the time, and

The rate at which water is *actually* available for use is best measured as the streamflow rate that is available a large percentage—say 95%—of the time.

One conceptually simple but highly informative way to summarize the variability of a time series is a cumulative frequency graph called a *duration curve*:

A **duration curve** is a graph showing the fraction (percent) of time that the magnitude of a given variable is exceeded.

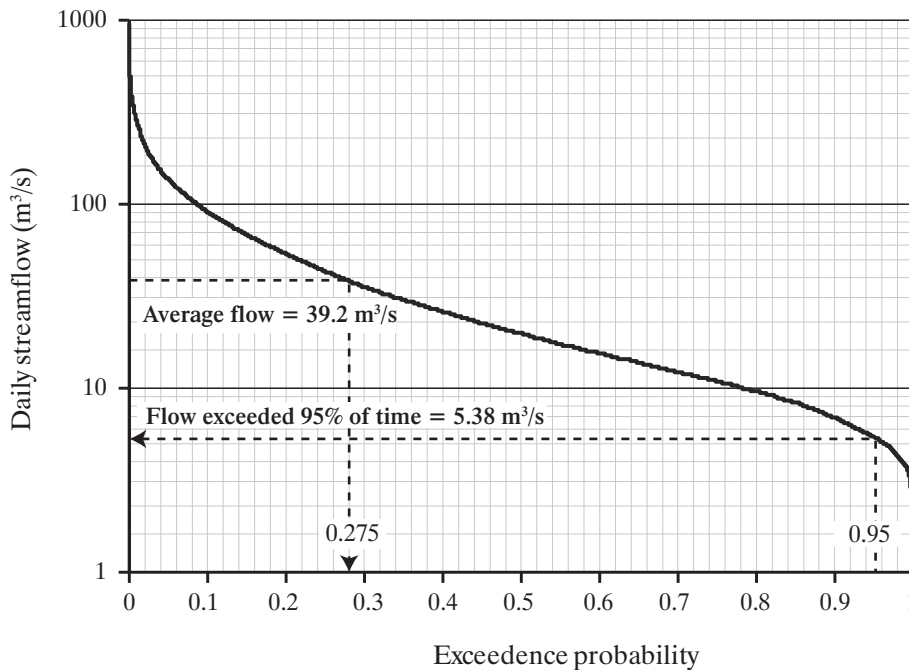
Duration curves are commonly used to depict the temporal variability of streamflow; such curves are called **flow-duration curves (FDCs)**. Figure 1.17 shows the FDC for the Pemigewasset River at Plymouth, New Hampshire, the same streamflow record analyzed in figure 1.16. To illustrate the significance of time variability, note that the average streamflow rate for the years plotted in figure 1.16 was 39.2 m<sup>3</sup>/s. The minimum and maximum daily average flows were 1.27 and 1,620 m<sup>3</sup>/s, respectively, so the ratio of maximum to minimum was 1,270! The rate that was exceeded 95% of the time was 5.38 m<sup>3</sup>/s, so the “available water resource” for this watershed is only about 14% of the average flow. Looked at another way, the streamflow equaled or exceeded the average only about 27.5% of the time. These values are typical of medium-sized humid watersheds; table 1.4

summarizes the general effects of watershed properties on natural streamflow variability.

The most widely used strategies for increasing water availability are (1) to decrease streamflow variability by building reservoirs (section 1.10.2) and (2) to extract water from natural ground-water storage reservoirs (chapter 9). One may also attempt to increase it by increasing  $\mu_P$  through “rain making” (section 4.4.4.2), or by modifying vegetation to reduce  $\mu_{ET}$  (section 6.6.5.2). However, all interferences in the natural hydrologic cycle usually have significant environmental, social, economic, and legal consequences.

**1.9.2.3 Nonstationarity**

Implicit in the usual computation of regional water balances and many other hydrological analyses is the assumption that there are no significant



**Figure 1.17** Flow-duration curve for the Pemigewasset River, New Hampshire. Note that the average flow, 39.2 m<sup>3</sup>/s, is exceeded only 27.5% of the time. The flow that is available 95% of the time is 5.38 m<sup>3</sup>/s, so the “available water resource” for this watershed is only about 14% of the average flow.

**Table 1.4** General Effects of Watershed Properties on Relative Time Variability of Natural Streamflow.

Property	Effect
Size	• Larger watersheds tend to have lower variability (storage effect).
Geology	• Watersheds underlain by porous formations (sand, limestone) tend to have lower variability. • Watersheds underlain by crystalline rock and/or clay tend to have higher variability.
Climate	• Watersheds in humid regions tend to have lower variability. • Watersheds in regions of highly seasonal precipitation or snowmelt or in arid regions tend to have higher variability.

long-term trends or cycles, i.e., that hydrologic time series are **stationary**.

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**Stationarity** is the assumption that a time series is a representative sample of long-term (i.e., future) behavior.

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If stationarity can be assumed, most traditional statistical concepts can be applied to time series, including the idea that increasing sample size (i.e., length of observation period) increases confidence in statistical inferences. Clearly, this idea is questionable if the underlying system is changing over time.

As pointed out by Milly et al. (2008), the stationarity assumption has long been compromised by human disturbances (e.g., water infrastructure, channel modifications, drainage works, land-cover and land-use changes) as well as natural climate changes and low-frequency internal variability (the *teleconnections* discussed in section 2.1.6). Traditionally, water-resource planners have considered natural change and variability to be sufficiently small to allow stationarity-based design, and have made adjustments for significant known human disturbances.

However, substantial anthropogenic change of the earth's climate is altering the means and extremes of precipitation, evapotranspiration, and rates of discharge of rivers (see section 2.2.8), and the relation between precipitation and streamflow (Zhang et al. 2011). Because of this, Milly et al. (2008, p. 573) assert that “stationarity is dead and should no longer serve as a central, default assumption in water-resource risk assessment and planning.” Development of a replacement for the concept is a daunting conceptual and practical problem for hydrologists. The solution will have to combine historical and paleohydrologic measurements with projections of improved climate and hydrologic models (e.g., Gilroy and McCuen 2012). Milly et al. (2008, p. 574) emphasize that

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“In a nonstationary world, continuity of observations is critical.”

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Statistical tests described in section C.8.5 can help detect nonstationarity in time series of climatic and hydrologic data.

## 1.10 Hydrologic Storage

### 1.10.1 Definition

In the global hydrologic cycle a given molecule of water is always in one of the storage components of figure 1.2, and in the land phase of the cycle a given molecule is always in one of the components of figure 1.3. Thus, although the term “storage” often connotes a static situation, water is always moving through any control volume and

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Water in the hydrologic cycle is always in motion  
AND always in storage, and any hydrologic  
control volume represents storage.

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### 1.10.2 Storage Effects

In many hydrologic reservoirs, such as lakes, segments of rivers, ground-water bodies, and watersheds, the outflow rate increases as the amount of storage increases.<sup>6</sup> For these situations, we can model the relation between outflow rate,  $\vartheta$ , and storage volume,  $S$ , as

$$\vartheta = f_{\vartheta}(S). \quad (1.23)$$

In some cases, the nature of the **outflow function**  $f_{\vartheta}(S)$  can be developed from the basic physics of the situation. In most hydrologic control volumes, such as natural watersheds, equation (1.23) is merely a conceptual model. In the model of watershed functioning described in section 1.12, observations of  $\vartheta$  and  $d\vartheta/dt$  are used to deduce the form of  $f_{\vartheta}(S)$ .

The simplest version of equation (1.23) describes a **linear reservoir**:

$$\vartheta = k_R \cdot S, \quad (1.24)$$

where  $k_R$  is a positive constant. Although no natural reservoir is strictly linear, equation (1.24) is often a useful approximation of hydrologic reservoirs (section 10.2.5). Furthermore,  $k_R$  has a simple physical interpretation in a linear reservoir: Since  $\vartheta$  and  $S$  have the dimensions of  $[L^3 T^{-1}]$  and  $[L^3]$ , respectively, the dimensions of  $k_R$  are  $[T^{-1}]$  and, as will be shown in section 1.10.3,  $1/k_R$  is a significant reservoir time constant.

Where equation (1.23) applies, storage has two effects on outflow time series:

1. Storage decreases the *relative variability* of the outflows relative to the inflows. Standard statistical



measures such as the coefficient of variation (ratio of standard deviation to mean; section C.3.5) or simple ratios determined from FDCs, such as  $Q_{95}/\mu_Q$ , can be used to quantitatively characterize relative variability.

- Storage increases the *persistence* of the outflows relative to the inflows, i.e., the tendency for high values to be followed by high values, and low values by low values.

As explained in section C.6.2, persistence can be characterized by the *autocorrelation coefficient* of a time series.

As noted earlier, equation (1.23) applies to most watersheds, and table 1.4 lists some watershed characteristics associated with varying degrees of outflow variability. And, of course, water-supply reservoirs are built to increase the available water resource by decreasing outflow variability.

### 1.10.3 Residence Time

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The **residence time** of a reservoir is the average length of time that a “parcel” of water spends in the reservoir.

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Residence time (also called **transit time** or **turn-over time**) is a universal relative measure of reservoir storage (Bolin and Rodhe 1973). For a reservoir in steady state (which can usually be assumed for natural hydrologic reservoirs), the average rates of inflow and outflow are equal, and

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Residence time,  $T_R$ , can be calculated by dividing the average reservoir storage,  $\mu_S$  ([M] or [L<sup>3</sup>]), by the average rate of inflow,  $\mu_I$ , or outflow,  $\mu_O$  ([M T<sup>-1</sup>] or [L<sup>3</sup> T<sup>-1</sup>]).

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That is,

$$T_R = \frac{\mu_S}{\mu_I} = \frac{\mu_S}{\mu_O}. \quad (1.25)$$

For many hydrologic reservoirs, such as lakes, values of  $\mu_S$  and  $\mu_I$  or  $\mu_O$  can be readily determined, and computation of residence time is straightforward. For others, such as watersheds and ground-water bodies, it may be difficult to determine the value of  $\mu_S$  with precision; in these cases, we can usually speak of residence times in relative terms—for

example, under similar climatic regimes, streams receiving water from ground-water reservoirs with large residence times (i.e., with large volumes of storage per unit watershed area) will tend to have less variable and more persistent streamflow than those receiving water from reservoirs with shorter residence times (table 1.4).

For a linear reservoir, equation (1.24) applies to the average values, i.e.,

$$\mu_O = k_R \cdot \mu_S \quad (1.26)$$

and we see from (1.25) that the reservoir constant  $k_R$  equals the inverse of the residence time:

$$k_R = \frac{1}{T_R}. \quad (1.27)$$

We can quantify the variability and persistence effects of storage (section 1.10.2) as functions of residence time for a linear reservoir, as shown in figure 1.18. Note that the effects are substantial, even for small values of  $\mu_S/\mu_O$ .

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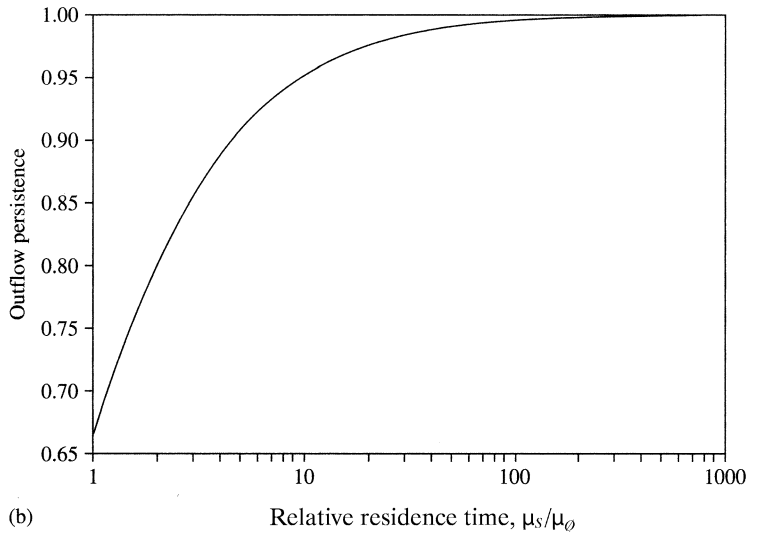
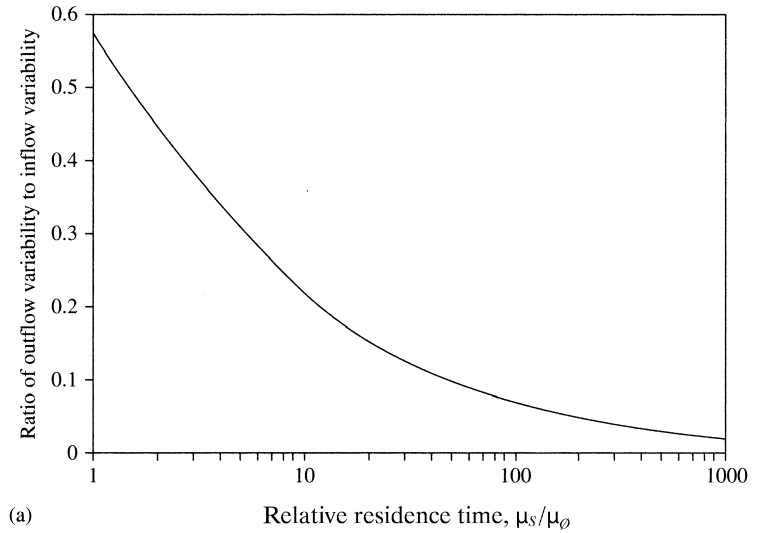
## 1.11 Uncertainty in Hydrology

### 1.11.1 Causes of Uncertainty

Hydrology is a highly uncertain science due to natural variability and to lack of knowledge (Montanari et al. 2009). Some of the specific causes of this uncertainty are:

- Many hydrologic processes are inherently random (weather processes, subsurface flow paths).
- Hydrologic processes in a given region may be affected by processes occurring in distant regions and/or at earlier times.
- Although the physics of many important processes are known at the “microscopic” scale, we do not completely understand the dynamics of these processes at the watershed and larger scales at which they are observed and modeled.
- We cannot observe in detail, and thus cannot mathematically represent, the geometry of hydrologic control volumes (soils, aquifers, river beds, watersheds), as well as most of the initial and boundary conditions.
- Hydrologists typically work under conditions of data scarcity.

As discussed in detail in appendix F, computer models simulating the functioning of the hydrologic



**Figure 1.18** (a) Ratio of relative variability of outflows to relative variability of inflows as a function of relative residence time  $\mu_s/\mu_\phi$  for a linear reservoir [equation (1.24)]. (b) Persistence of outflows (expressed as the autocorrelation coefficient) as a function of relative residence time for a linear reservoir when inflows have no persistence.

cycle are widely used as tools in scientific investigations, as means of making real-time forecasts of natural hazards, and for assessing longer-term effects of climate change or proposed water-resource infrastructure. In all applications, quantifying the uncertainty of the model output response is extremely important, and uncertainty assessment is receiving increasing attention in hydrologic research (Montanari et al. 2009). In the remainder of this section, we discuss a more limited but equally important aspect of uncertainty: the assessment of uncertainty in measured quantities and its propagation in computations.

### 1.1.1.2 Treatment of Random Uncertainty in Computations

Every measurement, no matter how precisely and carefully made, is made with some degree of uncertainty about the difference between the result of the measurement and the “true” value of the measured quantity. Consequently, any computation made with measured quantities is also subject to uncertainty. Thus,

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Understanding uncertainty and how it propagates through computations is an essential aspect of science.

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### 1.11.2.1 Systematic and Random Error

The result of a measurement of a quantity  $x$  is expressed as

$$x_{meas} = x \pm \delta x, \quad (1.28)$$

where  $x_{meas}$  is the measured value and  $x$  is the “true” value. The difference,  $\delta x$ , is the **error**: the upper and lower bound of the range<sup>7</sup> in which the observer is “reasonably confident” that the true value lies.

Measurement errors are of two types: *systematic* and *random*.

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**Systematic error**, or **bias**, is due to inherent tendencies for instrumentation or methods of observation to consistently record values that are on average higher or lower than the “true” values.

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These biases are often difficult to detect, but can be quite serious in hydrology. For example, virtually all methods for measuring precipitation at a point significantly undermeasure the true value (see section 4.2.1). In fact, Adam and Lettenmaier (2003) estimated that global precipitation is undermeasured by about 12%. Detecting systematic errors is often difficult and requires careful analysis of the particular methodology involved and comparison with other methods, so there are no general approaches to accounting for them.

Thus, although one must always be aware that systematic errors may be present, the discussion here deals only with random errors:

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**Random error** results from unpredictable fluctuations in values obtained in a given measurement situation that are equally likely to be greater than or less than the true value. The likelihood of random error is inversely reflected in the **precision** of a given measurement.

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### 1.11.2.2 Absolute and Relative Error

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In equation (1.28),  $\delta x$  is called the **absolute error**.

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In hydrology, uncertainty is often expressed as a relative value,  $\varepsilon$ , where

$$\varepsilon \equiv \frac{\delta x}{x_{meas}}. \quad (1.29)$$

---

$\varepsilon$  is called the **relative error**.

---

It is virtually never possible to know with 100% assurance the range in which the true value lies, so the confidence in a given measurement is usually specified as:

$$\text{“I am } 100 \cdot p \text{ \% sure that } x_{meas} - \delta x \leq x \leq x_{meas} + \delta x\text{”} \quad (1.30)$$

or

$$\text{“I am } 100 \cdot p \text{ \% sure that } x_{meas} - \varepsilon \cdot x_{meas} \leq x \leq x_{meas} + \varepsilon \cdot x_{meas}\text{”} \quad (1.31)$$

where  $0 \leq p < 1$ . Obviously the values of  $p$  and  $\delta x$  or  $\varepsilon$  are related:

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$\delta x$  and  $\varepsilon$  increase as  $p$  increases for a given measurement situation.

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Appendix A presents the concepts of precision and significant figures, and these concepts must be kept in mind when stating uncertainties. As noted there, most hydrological measurements are made with no more than 3-significant-figure precision, and

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Uncertainty  $\delta x$  cannot be stated with a greater *absolute* precision than the measured value.

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For example, if discharge is reported as 32.5 m<sup>3</sup>/s with a relative precision  $\varepsilon = 10\%$ , the uncertainty  $\delta x$  must be given as 3.2 m<sup>3</sup>/s, not 3.25 m<sup>3</sup>/s.

### 1.11.2.3 Statistical Characterization of Random Error

The central theory of random errors is called the **Gaussian theory of errors**. It is based on the assumptions that (1) the underlying processes generating the observed values are unchanged during the measurements; (2) we are at least conceptually able to make a large number  $N$  of repeated measurements of a quantity  $x$  using the same methodology; and (3) each measurement is subject to a number of sources of potentially compensating positive and negative